Self-starting passive mode locking

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We investigate the evolution of continuous-wave laser oscillation from free-running to mode-locked operation assuming a nonlinear device with an intensity-dependent transmittivity or reflectivity to be the mode-locking element. An intensity threshold for self-starting passive mode locking is predicted and related to the linewidth of the first beat note of the power spectrum of the free-running laser output. Experimental results confirm the predictions of the theory.

Passive mode locking of cw lasers has been a reliable technique for subpicosecond and femtosecond optical pulse generation. A common feature of these lasers is the existence of a threshold intracavity power required for self-starting mode locking. Laser media with short relaxation times (dyes, color centers) can efficiently participate in the pulse-shaping process themselves and therefore have low thresholds when mode locked with slow saturable absorbers. By contrast, solid-state laser materials with long upper-state lifetimes are incapable of taking part in pulse formation and necessitate fast-response nonlinear devices for mode locking. Although several nonlinear elements other than the conventional fast saturable absorber have been devised,[1–4] until recently passive mode locking could be demonstrated only in pulsed systems because of the high intensities needed to produce sufficient nonlinearity for coupling the axial modes. The exploitation of the Kerr nonlinearity of a single-mode optical fiber inserted in a coupled cavity has resulted in the first successful cw passive mode locking of a solid-state laser.[5] The technique has been termed additive-pulse mode locking[6] (APM). Owing to the tremendous enhancement in the efficiency of the nonlinear processes in optical fibers, self-starting APM has been easily achieved at intracavity powers as low as a few hundred milliwatts.[7,8] More recently, self-starting cw passive mode locking was also achieved with a bulk nonlinearity in a high-power laser.[9]

One of the most important questions in the development of new passive mode-locking techniques and in the design of cw passively mode-locked solid-state lasers is the cw intracavity power needed for the laser to become mode locked with a given nonlinear device or, conversely, the necessary nonlinearity that has to be introduced in a cw laser for self-starting mode locking with the available intracavity power. The purpose of this Letter is to derive a condition for self-starting cw passive mode locking of solid-state lasers by using a nonlinear element with an intensity-dependent transmittivity or reflectivity. We investigate the evolution of laser oscillation from the free-running state toward the steady-state mode-locked operation. Since the intensity-induced change in the round-trip loss is low during most of the transient pulse evolution, it may be expressed as a linear function of the intracavity intensity as long as its response time is short compared with the mode-locked pulse width. This approximation permits a unified characterization of different nonlinear devices and thus a unified treatment of the transient mode-locking process.

The evolution of the circulating radiation in a homogeneously broadened laser is governed by the equation[10]

$$\frac{\partial}{\partial T}v(t, T) = \frac{1}{2T_R}\left(g - l - q + D\frac{\partial^2}{\partial t^2}\right)v(t, T), \qquad (1)$$

where $v(t, T)$ is the electric-field amplitude of the circulating electromagnetic wave at a fixed position on the cavity axis as a function of the fast time parameter $t(0 \le t < T_R)$ and the slow time parameter $T(T \gg T_R)$. T_R is the round-trip time, l and g are the round-trip intensity loss and gain, respectively, q describes the effect of nonlinearity employed for pulse forming, and the differential operator accounts for dispersion. D represents the bandwidth limitation due to the finite gain bandwidth and other possible bandwidth-limiting elements.

The mode-locked laser pulse will evolve from the most intensive mode-beating fluctuation present in the multimode free-running laser. Neglecting the smaller fluctuations, we can write the circulating photon flux as $|v|^2 = S(T) + s(t, T)$, where $S(T)$ is the slowly varying average background intensity and $s(t, T)$ is the most intensive mode-beating fluctuation in the cavity. The intensity is normalized to the photon energy $\hbar\omega$ and is measured in the gain medium. The nonlinearity used for mode locking[1–9] causes an intensity-dependent change in the cavity round-trip loss, which is, to first order, given by

$$q(t, T) = -\kappa s(t, T). \qquad (2)$$

The S-dependent part of q is incorporated in l. Equation (2) applied to all types of passive absorberlike mode locker until the intensity becomes high and/or the pulse duration becomes comparable with the response time of the nonlinearity. κ is proportional to A_g/A_q, where A_g and A_q represent the effective beam cross sections in the gain and nonlinear medium, re-

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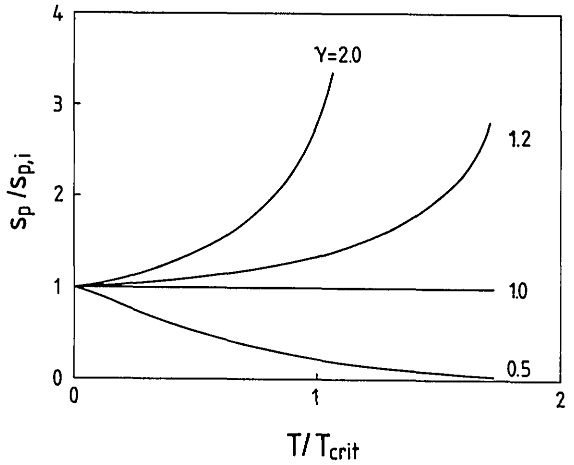

Fig. 1. Evolution of the normalized peak intensity $s_p/s_{p,i}$ as a function of the slow time parameter T for different values of $\gamma = \tau_c/T_{crit}$ at a fixed T_{crit}.

spectively. Sometimes it is useful to introduce $\kappa' = \hbar\omega A_g\kappa$, which is characteristic of the nonlinearity alone and relates q to the circulating intracavity power. In typical APM lasers using an $\approx$1-m length of single-mode fiber, $\kappa' \approx 10^{-4}$ W^{-1}. As a rule, κ' is orders of magnitude smaller for bulk nonlinearities.

Instead of a complete characterization of the evolution of passive mode locking, we restrict ourselves to following the development of the peak intensity $s_p(T) = s(0, T)$ of the fluctuation. This parameter is a good measure of the evolution of mode locking even in the last stage when the pulse energy $E_p(T)$ is approximately constant. From Eqs. (1) and (2) we can obtain after some manipulations the rate of the peak intensity growth of the perturbation,

$$\frac{\mathrm{d}s_p}{\mathrm{d}T} = \frac{1}{T_R}\left[\left(\kappa - \frac{1}{2}\alpha\sigma g_0\tau_p\right)s_p - \beta\frac{4D}{\tau_p^{\,2}} - l_0\right]s_p, \qquad (3)$$

where $\tau_p(T)$ is the pulse duration and $\alpha = s_p\tau_p/E_p$ and β are pulse-shape-dependent numerical factors of order unity. For a Gaussian pulse shape, $\alpha = 1.06$ and $\beta = 0.70$. The term containing the stimulated-emission cross section σ accounts for dynamic gain saturation.[11] For brevity, we introduced $l_0(T) = l - g_0$, where g_0 is the quasi-stationary gain. If we assume an absence of relaxation oscillations, the average intensity is approximately constant during the pulse evolution, i.e., $T_R(\mathrm{d}S/\mathrm{d}T) + \mathrm{d}E_p/\mathrm{d}T = 0$. In the early stage of the transient mode-locking process $E_p \ll T_R S$, and therefore $l_0 \approx 0$. The incipient growth of s_p is not affected by gain dispersion, because the initially oscillating modes experience the same round-trip gain. Thus $\mathrm{d}s_p/\mathrm{d}T > 0$ if $\kappa > \alpha\sigma g_0\tau_{p,i}/2$ $[\tau_{p,i} = \tau_p(0)]$, a condition similar to that obtained for $\mathrm{d}E_p/\mathrm{d}T > 0$ by other authors recently.[11] When dynamic gain saturation is negligible, Eq. (3) can be solved, and we find that $s_p \to \infty$ as $T \to T_{crit}$, where $T_{crit} = T_R/\kappa s_{p,i}$ and $s_{p,i} = s_p(0)$. The reaching of this point is prevented by the onset of dispersion owing to the shortened pulse duration τ_p at high peak intensities.

According to these results, mode locking is, regardless of the laser intensity, self-starting if the dynamic gain saturation is small enough compared with the nonlinearity. This is in contradiction with experimental observations. The insufficiency of Eq. (3) becomes obvious in the absence of nonlinearity and dynamic gain saturation (κ, $\sigma = 0$). For this case Eq. (3) predicts that s_p = const., which implies that the ratios of the complex mode amplitudes of the oscillating modes $V_k(t)/V_l(t)$ should be constant. In reality, however, the axial modes oscillating in the free-running laser have a finite correlation time τ_c. During this characteristic time the cavity modes tend to lose mutual coherence by random perturbations of the phase differences $(\phi_k - \phi_l)$ and/or the ratios of amplitudes $|V_k/V_l|$. Recalling that s_p is the most intensive fluctuation in the cavity, i.e., it originates from dominantly constructive interference of cavity modes, we conclude that s_p will decay with a characteristic lifetime τ_c. Simultaneously, an intensive fluctuation arises at another instant within the time interval $0 \le t < T_R$. We assume $s_p(T)$ to decrease exponentially with a time constant τ_c in the absence of nonlinearity. This corresponds to an additional term having the form $-s_p/\tau_c$ on the right-hand side of Eq. (3). This approximation may be justified by the ability of the model to predict experimental results. The fundamental equation governing the development of s_p at the early stage of the transient mode-locking process can now be written as

$$\frac{\mathrm{d}s_p}{\mathrm{d}T} = \frac{\kappa}{T_R}s_p^{\,2} - \frac{s_p}{\tau_c}, \qquad (3')$$

where dynamic gain saturation is neglected. The solution of Eq. (3′) is given by

$$s_p(T) = \frac{s_{p,i}}{\gamma + (1-\gamma)\exp(T/\tau_c)}, \qquad (4)$$

where $\gamma = \tau_c/T_{crit}$. Figure 1 shows $s_p(T)$ for different values of the parameter γ. $s_p(T)$ goes to zero for $\gamma < 1$, and $s_p(T) \to \infty$ for $\gamma > 1$. Thus self-starting mode locking requires that $\gamma > 1$. The development of $s_p(T)$ in this case is shown on a larger scale in Fig. 2. The dashed curves are hypothetical representations of the

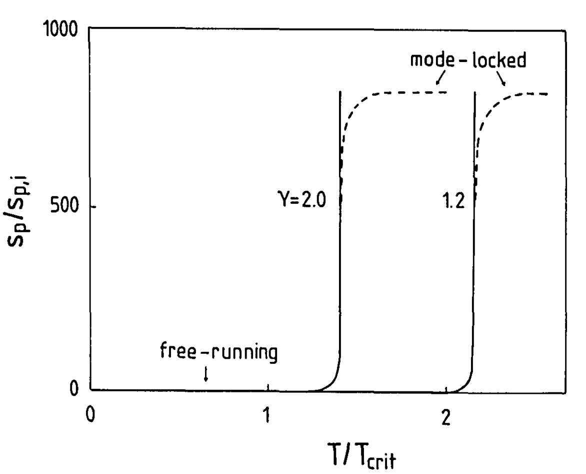

Fig. 2. Development of $s_p/s_{p,i}$ for different values of γ (>1) at a fixed T_{crit}. The dashed curves represent the effect of dispersion due to bandwidth limitation.

effect of dispersion at the end of the pulse evolution. With an initial number of modes m_i, the peak intensity of the most intensive fluctuation in the cavity is given by $s_{p,i} \approx S_i \ln(m_i)$,[12] where $S_i = S(0)$ is the average photon flux in the free-running laser. Making use of this relationship, we can write the condition for self-starting as

$$\kappa' P_i > \frac{1}{\ln(m_i)} \frac{T_R}{\tau_c}, \quad (5)$$

where P_i is the circulating intracavity power in the free-running laser. Note that increasing m_i does not necessarily reduce the self-starting threshold because it generally gives rise to a reduction of τ_c.

Quantitative prediction of the threshold for $\kappa' P_i$ calls for the knowledge of τ_c. In order to obtain a rigorous definition of τ_c we introduce the correlation function

$$G_{kl}(\tau) = \langle V_k(t) V_l^*(t) V_k^*(t+\tau) V_l(t+\tau) \rangle \cos \omega_{kl}\tau, \quad (6)$$

where the angle brackets denote time average. Furthermore $\omega_{kl} = |k-l|\Delta\omega_0 + \delta\omega_{kl}$, where $\Delta\omega_0$ is the mode spacing at the line center and $\delta\omega_{kl}$ is some nonlinear frequency shift that depends on k and l rather than only on their difference due to nonlinear (quadratic or higher order) dispersion inside the cavity. The power spectrum of the laser output at the nth beat note is given by

$$P_n(\omega) = \frac{1}{\pi} \sum_{k-l=n} \int_{-\infty}^{+\infty} G_{kl}(\tau) \exp(i\omega\tau) d\tau. \quad (7)$$

Cavity length fluctuations do not perturb the initial fluctuation, i.e., they do not affect τ_c, but they do cause random shifts of $P_n(\omega)$, the magnitude of which is proportional to n. It is the smallest and usually negligible at $n = 1$, the first beat note. Therefore we use $P_1(\omega)$ for the definition of τ_c. If we assume $P_1(\omega)$ to have a Lorentzian line shape, the sum of the cross-correlation functions of the adjacent axial modes decays exponentially with a time constant, which we identify with τ_c and is given by

$$\tau_c = \frac{1}{\pi} \frac{1}{\Delta\nu_{3\,\mathrm{dB}}}, \quad (8)$$

where $\Delta\nu_{3\,\mathrm{dB}}$ is the 3-dB full width of the first beat note. Beat note line shapes other than Lorentzian result only in a slight change in the proportionality factor $1/\pi$.

Relations (5) and (8) now offer the possibility of testing our simple model by measurements. First, the general trends predicted by relation (5) have been investigated by using an APM Nd:glass laser. We found, in agreement with the results of others,[7] that a reduced κ' requires a higher P_i for self-starting and vice versa. A decrease in T_R definitely reduced the intensity needed for self-starting at a fixed κ'. In the Nd:phosphate glass laser with $T_R = 10$ nsec and $m_i \approx 1.5 \times 10^2$ we obtained 2.1 kHz $< \Delta\nu_{3\,\mathrm{dB}} <$ 3.5 kHz for different cavity parameters.[13] The self-starting threshold is predicted to take values of $1.3 \times 10^{-5} < (\kappa' P_i)_{\mathrm{th}} < 2.2 \times 10^{-5}$. Direct measurements of the threshold power yielded $2.2 \times 10^{-5} < (\kappa' P_i)_{\mathrm{th}} < 3.3 \times 10^{-5}$. In considering experimental uncertainties and the simplicity of the model, the predictions of the above analysis appear to be in good qualitative and quantitative agreement with experiments. Note that any kind of phase modulation has been left out in the approach presented. Actually, if the nonlinearity causes some chirp to accumulate on the circulating pulse, significant group-velocity dispersion in the cavity may considerably influence the transient pulse evolution. By careful design, one can take advantage of these additional pulse-shaping effects.

The presented condition for self-starting passive mode locking is not only an interesting general relationship between various laser parameters but also a useful tool in the design and development of cw passively mode-locked solid-state lasers. Simple spectral measurements of the free-running laser output permit predictions of the necessary amount of nonlinearity for a particular laser or the power requirements when a given nonlinearity is used. Profound understanding of the origins of the finite beat note linewidth necessitate further experimental and theoretical investigations. The results of this future research might be especially useful for cw passive mode locking of low- and moderate-power (e.g., diode-pumped) solid-state lasers.

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