Mode locking in solitary lasers

T. Brabec, Ch. Spielmann, and F. Krausz

Technische Universität Wien, Abteilung für Quantenelektronik und Lasertechnik, Gusshausstrasse 27, A-1040 Wien, Austria

Received July 16, 1991

We present an analysis of passively mode-locked lasers in which pulse formation is dominated by the interplay between self-phase modulation and negative dispersion in separate cavity elements. Steady-state pulse parameters and stability issues are discussed. Stability in these solitary systems relies on some passive amplitude modulation, and the ultimate system performance is found to depend sensitively on the magnitude of amplitude modulation relative to that of phase modulation.

Self-phase modulation (SPM) that arises from a fast Kerr nonlinearity, in conjunction with appropriate dispersive compensation, may result in shortening of intensive optical pulses. This solitonlike pulse shaping, which is purely a phase effect, can efficiently support ultrashort-pulse formation in lasers passively mode locked by slow or fast saturable absorbers. The comprehensive theory of Martinez *et al.*[1] provides an adequate description of a wide range of cw passively mode-locked (PML) lasers incorporating either of the two types of passive amplitude modulation (PAM) and solitonlike shaping. The analytical solution found by Martinez *et al.* has been used recently for the study of fast absorber systems.[2,3] These analytical approaches[1–3] are all based on the assumption that the fractional change in pulse shape during the passage through the cavity elements is small compared with unity. This assumption is almost always fullfilled by PAM but may not be met by phase-affecting cavity elements under certain conditions. In this Letter we analyze cw passively mode-locked lasers in which femtosecond pulse formation is dominated by strong solitonlike shaping rather than by PAM.

This research was stimulated by the recent appearance of a new class of PML femtosecond laser based on solid-state gain media. The high intracavity power and the long optical path within the amplifier medium (compared with those of dye lasers) lead to strong SPM in these systems. A breakthrough in solid-state ultrafast technology was achieved by exploiting solitonlike shaping mechanisms with the addition of negative group-velocity dispersion[4–9] (GVD). A common feature of these systems is that the phase of the mode-locked pulse is substantially changed during one transit of the resonator, therefore the approximation used in the establishment of the theories in Refs. 1–3 is not justified. A correct theoretical treatment of PML lasers in which mode locking is dominated by solitonlike shaping requires the pulse-shaping elements to be described by operators that do not commute. As a consequence, the equation of motion for the mode-locked pulse is an operator equation that cannot be transformed to a common differential equation as in Refs. 1–3. A schematic diagram of this novel generation of femtosecond lasers, which we call solitary lasers, is shown in Fig. 1. In addition to SPM and negative dispersion, PAM has been included in our model. PAM is always needed in PML lasers to build up the mode-locked pulses and keep the steady state stable against noise perturbations. With the assumption of a broadband laser medium (e.g., Ti:sapphire), gain dispersion has been left out in the present analysis. This approximation allows for an instructive comparison between the steady state of a discrete solitary system and the fundamental soliton in an optical fiber, where SPM and negative GVD are continuously distributed along the optical path.

A transfer operator $\hat{T}$ can be assigned to each position (z_1, z_2, z_3, z_4) of the solitary system. $\hat{T}$ governs the pulse evolution,

$$v_{n+1}(t) = \hat{T}v_n(t), \tag{1}$$

where $v_n(t)$ is the complex electric-field amplitude after the nth round trip and t is the local time retarded by $n\tau_D$ with respect to the real time, with τ_D the group delay on one round trip. For the position z_2, $\hat{T}$ can be written as

$$\hat{T} = e^{\hat{D}}e^{\hat{A}/2}e^{\hat{N}}e^{\hat{A}/2}, \tag{2}$$

where $\hat{D} = iD/2\mathrm{d}^2/\mathrm{d}t^2$, $\hat{N} = i\phi|v|^2$, and $\hat{A}$ stands for PAM. D denotes the group delay dispersion of the dispersive element, $\phi|v|^2$ is the round-trip nonlinear phase shift in the Kerr medium, and $|v|^2$ is normalized to give the power. A solitary system can now be defined by $D/\phi < 0$ and $|\hat{A}v| \ll |\hat{N}v|$. Both conditions are satisfied by femtosecond solid-state lasers.[4–9] The amplitude modulation is assumed to be introduced by an ideal fast saturable absorber, i.e., $\hat{A} = g - l + \kappa|v|^2$, where $g - l$ is the resultant time-independent round-trip gain, and $\kappa|v|^2$ accounts for the amplitude nonlinearity. It is important to notice that the steady-state pulse parameters in the solitary system are not sensitive to the specific choice of $\hat{A}$. The gain g is adjusted to conserve pulse energy, which is an input parameter in our model. The steady-state solution of Eq. (1) can be found iteratively by computing $\hat{T}^n v_0(t)$, where the iteration has to be repeated until $v_{n+1}(t) = \exp(i\varphi)v_n(t)$ and φ is a constant phase shift. The

0146-9592/91/241961-03$5.00/0

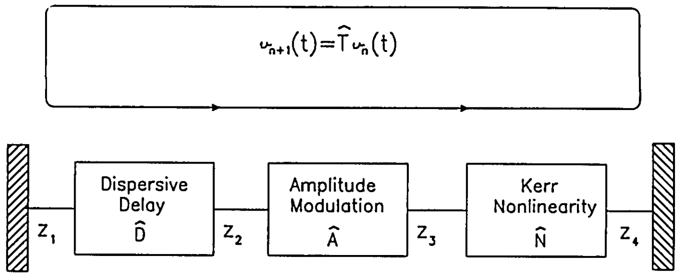

Fig. 1. Schematic of the solitary laser.

steady-state pulse at positions other than z_2 can be inferred from that at z_2 by applying the corresponding exponential operators.

Before evaluating our computer simulations it is expedient to quantify the difference between a soliton and solitary system. For this purpose we introduce the distributed parameters $\beta_2 = D/L_r$ and $\gamma = \phi/L_r$, where L_r is the round-trip length of the resonator. These parameters define a soliton-supporting system[10] that the solitary system goes into as the pulse energy approaches zero. A good measure for the discreteness of the solitary system is the ratio of L_r to the soliton period[10] $L_s = (\pi/2)(T^2/|\beta_2|)$, where $T = 2|\beta_2|/\gamma W$ and W is the pulse energy. Making use of these expressions and the definitions of β_2 and γ, we arrive at

$$r = \frac{L_r}{L_s} = \frac{1}{2\pi}\frac{(\phi W)^2}{|D|}. \qquad (3)$$

This parameter plays a fundamental role in the behavior of a solitary laser. Our investigations have revealed that unlike a soliton-supporting system ($r = 0$), a solitary system ($r > 0$) cannot support stable pulses without PAM even in the absence of noise perturbations. This is because the pulse is subjected to SPM without simultaneous GVD compensation, which results in a nonlinearly chirped pulse. A negative dispersion external to the nonlinearity compresses only the central portion of the pulse while giving rise to splitting at the pulse tails, where the chirp is nonlinear. PAM is able to prevent instabilities by pushing the portions that tend to split off toward the middle of the pulse. An increased r parameter requires stronger PAM for the pulse to remain stable. Conversely, ideal soliton formation is achieved in the limits $r \to 0$ and $\hat{A} \to 0$.

The most important characteristic of a mode-locked system, the pulse width, is determined by D, ϕ, and W in a solitary laser. As long as $|\hat{A}v| \ll |\hat{N}v|$, PAM has a negligible influence on the pulse duration. The discrete pulse-shaping elements result in different pulse widths at different positions in the cavity. From a practical point of view, the pulse widths at z_1 and z_4 are the most interesting, because output coupling is usually achieved at these positions. $\tau(z_1)$ and $\tau(z_4)$ are plotted as a function of D for different pulse energies in Fig. 2. For reference, the durations $\tau_s = 3.53|D|/\phi W$ of the corresponding $N = 1$ solitons are also depicted. The numbers chosen for the calculations are typical for state-of-the-art solid-state femtosecond lasers, except for the large value of κ, which has been taken to achieve stable mode locking for $\tau < 50$ fs. Because the discreteness of solitonlike shaping is measured by the r parameter, the deviation of solitary pulse parameters from solitons can be interpreted in terms of r. To be more precise, we assume the ratio of the solitary pulse width τ to τ_s to be a function of r, i.e., $\tau = \tau_s f(r)$. For the reasons discussed above, we expect $f(r)$ to approach unity as $r \to 0$. Thus a Taylor expansion to first order in r yields for the duration of the solitary pulse $\tau = \tau_s(1 + \alpha' r)$, where α' is an expansion coefficient. By using the expressions for τ_s and r, τ can be cast in the form

$$\tau = \frac{3.53|D|}{\phi W} + \alpha\phi W, \qquad (4)$$

where $\alpha = 0.56\alpha'$, i.e., the soliton pulse width is just increased by an amount proportional to ϕW independently of D. In fact, $\tau - \tau_s$ is linearly proportional to the pulse energy as revealed by Fig. 2. Equation (4) is accurate to within 10% for $r < 10$ and $\kappa/\phi < 0.2$. α depends only on the position, and we obtained $\alpha(z_1) = 0.1$ and $\alpha(z_4) = 0.25$. The connection between $\tau(z_1)$ and $\tau(z_4)$ can be written as $\tau(z_4) - \tau(z_1) = \beta(\phi W)$, where $\beta = 0.15$. This difference may become significant in powerful solitary lasers and should be taken into account in the design of such systems.

In addition to the pulse duration, the pulse shape and chirp are important characteristics of mode-locked lasers. To obtain some quantitative information of these parameters, we have computed the time–bandwidth products $\tau\Delta\nu$ of the steady-state pulses for different values of κ at z_1 and those $(\tau\Delta\nu)_{bl}$ of the corresponding bandwidth-limited pulses, i.e.,

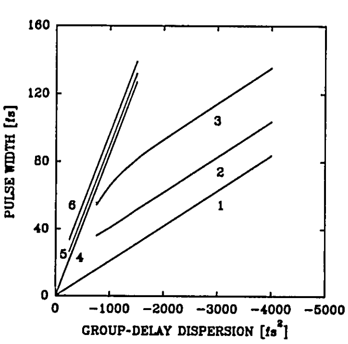

Fig. 2. Soliton and solitary pulse widths versus group-delay dispersion D for $\phi = 12.5 \times 10^{-7}\ W^{-1}$, $\kappa = 2 \times 10^{-7}\ W^{-1}$, and different pulse energies W. Curves 1, 2, and 3 illustrate τ_s, $\tau(z_1)$, and $\tau(z_4)$, respectively, for $W = 138$ nJ. Curves 4, 5, and 6 illustrate τ_s, $\tau(z_1)$, and $\tau(z_4)$, respectively, for $W = 34.5$ nJ.

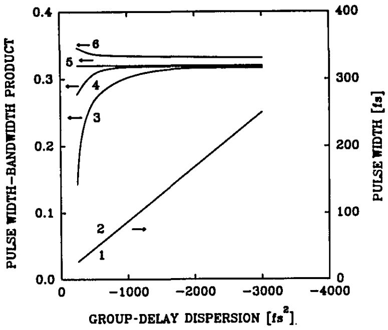

Fig. 3. Pulse width–bandwidth products $\tau\Delta\nu$ of solitary pulses and with the chirp removed $(\tau\Delta\nu)_{bl}$ for $\phi = 12.5 \times 10^{-7}\ W^{-1}$, $W = 34.5$ nJ, and different amplitude nonlinearities κ. Curves 1 and 2 denote τ; curves 3 and 4 denote $(\tau\Delta\nu)_{bl}$; curves 5 and 6 denote $\tau\Delta\nu$ for $\kappa = 2 \times 10^{-7}\ W^{-1}$ and $\kappa = 4 \times 10^{-7}\ W^{-1}$, respectively.

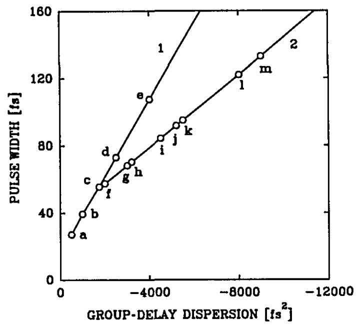

Fig. 4. Boundaries of regimes of stable mode locking on the $\tau(D, \phi W = \text{const.})$ curves for different amplitude and phase nonlinearities κ and ϕ. Curve 1, $\phi W = 172.5$ fs; curve 2, $\phi W = 345$ fs. The values of κ in units of $10^{-7}\ W^{-1}$ and the corresponding ratios κ/ϕ are as follows: a, $\kappa = 2$, $\kappa/\phi = 0.16$; b, 1, 0.08; c, 0.5, 0.04; d, 0.25, 0.02; e, 0.125, 0.01; f, 2, 0.16; g, 1, 0.08; h, 2, 0.08; i, 0.5, 0.04; j, 1, 0.04; k, 2, 0.04; l, 0.5, 0.02; m, 1, 0.02.

of pulses having the same shape and being free from phase modulation. $(\tau\Delta\nu)_{bl}$ provides information about the pulse shape, whereas $\tau\Delta\nu - (\tau\Delta\nu)_{bl}$ is a good measure of the chirp carried by the pulse. $(\tau\Delta\nu)_{bl}$ appears to decrease and $\tau\Delta\nu - (\tau\Delta\nu)_{bl}$ appears to increase with decreasing $|D|$ (see curves 3 and 5 in Fig. 3), which indicates that the pulse tails become longer and the chirp becomes greater for reduced negative dispersion. These results along with further investigations have shown that the pulse quality gets impaired in a solitary system as r increases. For $\kappa/\phi < 0.1$, solitary pulses significantly deviate both in shape and in phase from a bandwidth-limited hyperbolic-secant pulse once r becomes greater than 2. In this regime, increased PAM considerably improves the pulse quality by reducing both the wings and the chirp of the solitary pulses (curves 4 and 6 in Fig. 3). These results indicate that femtosecond solitary pulses must be subjected to a careful temporal and spectral analysis before making any statement about their shape and phase. Recently, femtosecond pulses with $(\tau\Delta\nu)_{bl} < 0.2$ have been generated in a high-power solitary laser.[11]

As expected, the gradual degradation of pulse quality with increasing r ends up in unstable operation when r reaches a critical value r_{crit}. This sets a limit to pulse shortening by reducing $|D|$. Figure 4 shows the minimum values of $|D|$ that allow stable mode-locked operation and the corresponding shortest pulse durations τ_{min} for different values of κ, ϕ, and W. Our simulations have revealed that r_{crit} is a sensitive function of κ/ϕ and only slightly dependent on ϕW. This implies that increased κ/ϕ not only improves pulse quality but also allows the generation of shorter pulses. Alternatively, shorter pulses can be produced by decreasing ϕW. As a consequence one may anticipate better femtosecond performance of a solitary system (free from bandwidth limitation and higher-order dispersion) at reduced pulse energies, but, clearly, W cannot be decreased without restriction because of the requirement of self-starting mode locking.[12] According to our results, stable sub-100-fs pulse generation in practical solitary systems with $\phi W \approx 100$ fs calls for $\kappa/\phi \approx 0.02$. This corresponds to a PAM of a few percent, in good agreement with experimental observations.[8]

In summary, we analyzed PML lasers in which pulse shortening is dominated by an interplay between a nonlinear phase shift in a Kerr medium and negative dispersion in a dispersive delay line. We introduced a parameter measuring the difference between a solitary and soliton system, which primarily determines the deviation of solitary pulses from solitons. Although pulse duration is not directly affected by amplitude modulation, the ratio of amplitude nonlinearity to phase nonlinearity has a strong influence on pulse quality and stability and sets an ultimate limit to pulse durations achievable in solitary systems.

This research was supported by the Jubiläumfonds der Österreichischen Nationalbank grant 3828.

References

1. O. E. Martinez, R. L. Fork, and J. P. Gordon, J. Opt. Soc. Am. B **2,** 753 (1985).
2. P. A. Bélanger, "Coupled-cavity mode-locking: a nonlinear model," J. Opt. Soc. Am. B (to be published).
3. H. A. Haus, J. G. Fujimoto, and E. P. Ippen, "Structures for additive-pulse mode locking," J. Opt. Soc. Am. B (to be published).
4. D. E. Spence, P. N. Kean, and W. Sibbett, Opt Lett. **16,** 42 (1991).
5. N. Sarukura, Y. Ishida, and H. Nakano, Opt. Lett. **16,** 153 (1991).
6. M. Hofer, M. E. Fermann, F. Haberl, M. H. Ober, and A. J. Schmidt, Opt. Lett. **16,** 502 (1991).
7. Ch. Spielmann, F. Krausz, T. Brabec, E. Wintner, and A. J. Schmidt, Appl. Phys. Lett. **58,** 2470 (1991).
8. L. Spinelli, B. Couillaud, N. Goldblatt, and D. K. Negus, in *Digest of Conference on Lasers and Electro-Optics* (Optical Society of America, Washington, D.C., 1990), paper CPDP-7.
9. Ch. Spielmann, F. Krausz, T. Brabec, E. Wintner, and A. J. Schmidt, Opt. Lett. **16,** 1180 (1991).
10. G. P. Agrawal, *Nonlinear Fiber Optics* (Academic, New York, 1989).
11. J. P. Likforman, G. Grillon, M. Joffre, C. Le Blanc, A. Migus, and A. Antonetti, Appl. Phys. Lett. **58,** 2061 (1991).
12. F. Krausz, T. Brabec, and Ch. Spielmann, Opt. Lett. **16,** 235 (1991).