Kerr lens mode locking

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Received May 28, 1992

Self-focusing in conjunction with an intracavity aperture creates a power-dependent amplitude modulation in laser oscillators, which allows passive mode locking. A simple analytical formalism yields closed-form expressions for the depth of passive amplitude modulation introduced by either the spatial gain profile or a hard aperture inserted in the resonator. Design issues for this mode-locking technique are discussed.

It has been known for a long time that intense laser beams are subject to self-focusing when they propagate through optical media that have a nonlinear index of refraction. This Kerr self-focusing effect leads to slight changes in the spatial intensity profile of the resonator mode in laser oscillators. As a consequence, by introducing an intracavity aperture, a power-dependent loss can be created. When carefully optimized, this passive amplitude-modulation mechanism favors a high intracavity power, i.e., mode-locked operation over free-running laser oscillation. Owing to the quasi-instantaneous response of nonresonant Kerr nonlinearities, the amplitude modulation induced by self-focusing is able to simulate ultrafast saturable-absorber action and support pulse formation down to the femtosecond regime in solid-state lasers that have long gain-relaxation times.[1–8] The technique has been termed self-mode-locking[1] or Kerr lens mode locking[2] (KLM). Recently computer simulations have verified the mechanisms responsible for KLM.[9–11] In this Letter a simple analytical description based on the self-similar solution of the nonlinear wave equation describing self-focusing[12,13] is presented. The derived results give insight into the basic physical processes in KLM and provide guidelines for the design and optimization of KLM lasers.

The analysis makes the following assumptions and approximations: (1) The resonator consists of two focusing mirrors (or lenses) and two flat end mirrors (Fig. 1). The Kerr medium is inserted into the tightly focused section of the cavity. The nonlinear medium is assumed to be plane cut, so that astigmatism can be neglected. (2) Only the fundamental Gaussian mode is oscillating in the resonator. (3) A self-similar solution of the nonlinear wave equation in the aberrationless approximation (quadratic index profile) is used for the description of propagation in the nonlinear medium. (4) Perturbations to the mode profile by the gain aperture or some additional hard apertures are negligible.

Figure 1 shows a schematic diagram of the resonator commonly used for KLM. The focal length of the lenses are f_1 and f_2, and d_1 and d_2 ($d_1 \leq d_2$) are the distances between the lenses and the output couplers. The spacing between the two lenses is $d_f = f_1 + f_2 + \delta$, where δ is the parameter defining the range of stable operation. Note that d_f is somewhat different from the real physical separation of the lenses in the presence of some dense optical medium in between. The position of the beam waist in the linear resonator can be obtained by determining the solutions of the equivalent two-mirror resonator[14] for which standard solutions exist. The radii of curvature of the mirrors and the length of the equivalent resonator are $R_1 = -f_1^2/(d_1 - f_1)$, $R_2 = -f_2^2/(d_2 - f_2)$, and $t = R_1 + R_2 + \delta$, respectively. If we assume that $|R_2| < |R_1|$, $d_1 > f_1$, and $d_2 > f_2$, stable operation for this configuration is possible for two separate ranges, namely for $0 < \delta < -R_2$ and for $-R_1 < \delta < -R_1 - R_2$. The different stability ranges can be quickly identified by comparison of the beam parameters at the resonator ends. The beam radius at the output coupler of the short arm is notably larger for the range $0 < \delta < -R_2$ than for $-R_1 < \delta < -R_1 - R_2$, whereas the radius at the output coupler of the long arm is comparable for both stability ranges.

In order to obtain high nonlinearity the Kerr medium has to be situated near the focal plane of the linear resonator. The optical length is l/n, where n is the linear refractive index, and lx_1/n and lx_2/n denote the distances between the focal point and the two surfaces of the Kerr medium ($x_1 + x_2 = 1$). The center of the nonlinear medium and the focal point coincide for $x_1 = x_2 = 1/2$. The self-similar solution of the propagation equation[12,13] implies that the nonlinearity modifies the confocal parameter of a Gaussian beam by a factor $1/\eta$, where $\eta = \sqrt{1 - P/P_{\mathrm{cr}}}$. Consequently the position of the beam waist x_{10} and confocal parameter $z_{r0} = \pi n w_{r0}^2/\lambda$ of the linear resonator mode are modified to $x_1(\eta)$ and $z_r(\eta)$. The vacuum laser wavelength is λ, n_2 is the nonlinear refractive index, and w_r is the radius (half-width at $1/e^2$ of the maximum intensity) of the beam waist. Here and throughout this Letter the subscript 0 refers to the solution of the linear resonator for $\eta = 1$. The critical power for self-trapping is $P_{\mathrm{cr}} = a\lambda^2/(8\pi n n_2)$, where a is an experimentally obtained correction factor[15] that extends the validity of the aberrationless model in the presence of a strong nonlinearity, and n_2 is the nonlinear refractive index. This expression yields for Ti:sapphire a power of 2.6 MW. Numerical investi-

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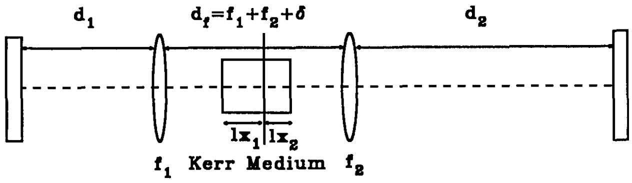

Fig. 1. Schematic of the cavity used in our calculations. The distances d_1 and d_2 and the length of the Kerr medium l are taken to be 80, 120, and 1.6 cm, respectively, throughout this Letter. The refractive index of the Kerr medium is assumed to have the value $n = 1.76$.

gations[16] have shown that owing to the inclusion of the correction factor the Gaussian self-similar description is valid for $\bar{P} = P/P_{cr} < 0.25$. From the discussion above, the radii of curvature of the phase fronts $\bar{R}_1$ and $\bar{R}_2$ at both surfaces of the Kerr medium can then be written as

$$\bar{R}_1(\eta) = \frac{l x_1(\eta)}{n}\left\{1 + \left[\frac{z_r(\eta)}{l x_1(\eta)\eta}\right]^2\right\}, \qquad (1)$$

where $\bar{R}_2$ is obtained by replacing the subscript 1 with 2 in Eq. (1). Outside the nonlinear index medium the Gaussian beam can be traced by the *ABCD* matrix formalism. Because of the reciprocity of the resonator mode the radii of curvatures of the mode are required to be infinite at the flat end mirrors. Reciprocity can be assumed as long as the nonlinearity only introduces a rescaling of the linear Gaussian propagation. From this requirement one obtains the following equations for the beam parameters in the nonlinear resonator:

$$\bar{R}_1^2(\eta) - \bar{R}_1(\eta)\left\{\bar{R}_{10} + \left[\frac{1}{x_{10}} - \frac{1}{x_1(\eta)\eta^2}\right]\frac{n}{l}\frac{B_1 D_1}{A_1 C_1}\right\} - \frac{1-\eta^2}{\eta^2}\frac{B_1 D_1}{A_1 C_1} = 0, \qquad (2)$$

$$\hat{T}_1 = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} \frac{f_1}{R_1} & \frac{f_1}{R_1}\left\{R_1 - \left[\frac{l x_{10}}{n} - \frac{\delta(\delta + R_2)}{2\delta + (R_1 + R_2)}\right]\right\} \\ -\frac{1}{f_1} & \frac{1}{f_1}\left[\frac{l x_{10}}{n} - \frac{\delta(\delta + R_2)}{2\delta + (R_1 + R_2)}\right] \end{bmatrix}. \qquad (3)$$

The equations for $\bar{R}_2$ and $\hat{T}_2$ can again be obtained by replacing 1 with 2. The matrices $\hat{T}_1$ and $\hat{T}_2$ relate the Gaussian beam parameters at the surfaces of the Kerr medium to the beam parameters at the output couplers. Combination of Eqs. (1) and (2) and the equivalent equations for side 2 of the resonator yields two equations for $w_r(\eta)$ and $x_1(\eta)$, which provide a complete description of the mode of the nonlinear resonator. Solution of the equations to first order in $\bar{P}$ yields the power-dependent change in position (Δx_1) and size (Δw_r) of the beam waist, which can be transformed into a power-dependent round-trip gain by employing some kind of aperturing in the cavity.

First, consider the case of aperturing the beam within the Kerr medium. In longitudinally pumped lasers the spatial gain profile can play the role of a soft aperture in the Kerr medium. For the evaluation of gain-based KLM the gain integral[17] has to be solved, where the beam waists and beam axes of resonator and pump beams are assumed to be collinear. For $z_p < z_{r0} < l/2$, $\delta = -R_2/2$, and $x_{10} = x_{20} = 1/2$, where z_p is the confocal parameter of the pump beam, a simple relation for the absolute change in power round-trip gain Δg can be derived:

$$\Delta g \approx \frac{g_s}{2}\frac{\bar{P}}{1+S}, \qquad (4)$$

where g_s is the saturated power round-trip gain of the linear resonator and $S = 2P_{av}/(\pi w_{r0}^2 I_s)$ is the gain saturation parameter, with P_{av} and I_s the average power and saturation intensity, respectively. As $\Delta g \rightarrow 0$ for large intracavity powers, the gain saturation limits the maximum obtainable amplitude modulation. This dependence is depicted in Fig. 2, where analytical (dashed curves) and numerical (solid curves) results of Δg versus $\bar{P}$ are plotted for different confocal parameters of pump and resonator beams for $P_{av} = 2$ W and $I_s = 300$ kW/cm^2. From Fig. 2, it can be seen that gain saturation reduces the efficiency of the amplitude modulation depending on the radius of the resonator mode. This reveals that for the design of a KLM system z_{r0} is an important parameter, and calculations have shown that a reasonable choice, combining a high gain and high modulation efficiency, is $z_{r0} < \approx l/2$. In the case of $z_p > l/2$, Δg becomes dependent on z_p, and the efficiency of the gain modulation is reduced. Furthermore KLM in combination with gain aperture yields amplitude modulation over most parts of the stability range, and optimum operation is achieved around the centers of both stability ranges.

Alternatively, a hard aperture can be introduced into one of the collimated resonator arms. For a low linear loss L introduced by the aperture inserted near one of the end mirrors the perturbations to the

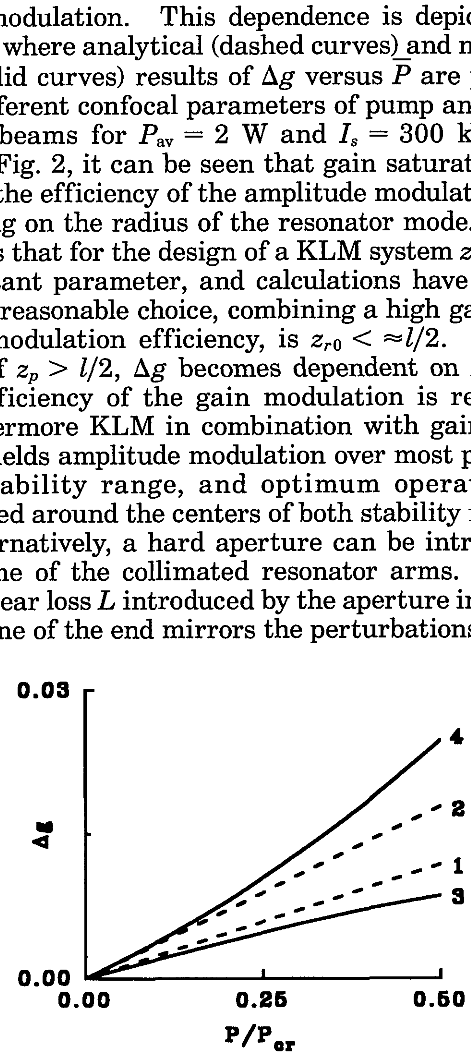

Fig. 2. Absolute change in gain Δg versus normalized intracavity power for a gain aperture within the Kerr medium. $g_s = 0.1$, $I_s = 300$ kW/cm^2, $P_{av} = 2$ W, and $\delta = R_2/2$. Curves 1 and 3 are the analytical and numerical solutions for $f_1 = f_2 = 6$ cm, $z_{r0} = 0.26$ cm, and $z_p = 0.22$ cm; curves 2 and 4 are the analytical and numerical solutions for $f_1 = f_2 = 10$ cm, $z_{r0} = 0.73$ cm, and $z_p = 0.22$ cm.

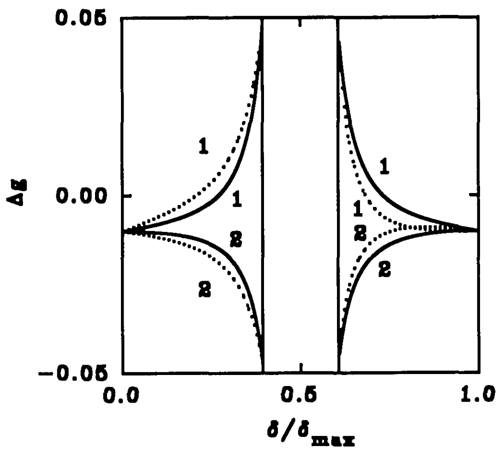

Fig. 3. Absolute change in gain versus the normalized stability parameter. $\delta_{\max} = R_1 + R_2$ and $L = 0.1$, $P = 0.25$, and $f_1 = f_2 = 6$ cm. The vertical lines denote the limits of the stability range. Curves 1 and 2 show the results for the aperture inserted at the output coupler of the short arm and of the long arm, respectively. The solid curves are for $\epsilon = 0$, and the dotted curves are for $\epsilon = 0.25$.

linear resonator mode may be assumed negligible and the nonlinear change in gain can be shown to be equal to $1 - [w_{1,2}^2(\eta)/w_{1,2}^2(\eta = 1)]$, where $w_{1,2}$ are the beam radii at the output couplers. By using the matrices $\hat{T}_{1,2}$ one can determine the variation of the radii w_1 and w_2 at the end mirrors from the change in the size and the position of the beam waist within the gain material. Hence the gain variation arising from an aperture in the short arm (index 1) and in the long arm (index 2) is given by

$$\Delta g_{1,2} \approx P \frac{nLR_{1,2}}{lP_{\mathrm{cr}}\left[1 + \left(\frac{l}{2z_{r0}}\right)^2\right]} \left\{ 1 \mp \frac{R_1^2 - R_2^2}{(R_1 + R_2 + 2\delta)^2} \pm \frac{8l\epsilon}{n\left[1 + \left(\frac{l}{2z_{r0}}\right)^2\right]} \frac{1}{R_1 + R_2 + 2\delta} \right\}, \tag{5}$$

where ϵ is defined by $x_{10} = 1/2 + \epsilon$, which denotes a possible shift of the nonlinear medium. Positive ϵ means a shift of the nonlinear medium toward the shorter resonator arm. From relation (5), which is plotted in Fig. 3 for a given set of resonator parameters, the most important features of hard-aperture KLM can be obtained immediately. Mode-locked operation depends mainly on three parameters, namely, the position of the Kerr medium (ϵ), the confocal parameter (z_{r0}), and the position in the stability range. Relation (5) reveals that for a large confocal parameter high amplitude modulation can be achieved. However, if z_{r0} becomes greater than $l/2$ this results in a reduction of the gain, which decreases the maximum loss bias that can be produced by the hard aperture. A reasonable trade-off can be realized by choosing $z_{r0} \approx l/2$. As can be seen from relation (5) and the curves labeled 1 in Fig. 3, mode-locked operation ($\Delta g > 0$) for $\epsilon = 0$ can only be obtained with the aperture in the short resonator arm. Figure 3 demonstrates that enhanced gain modulation can be obtained by shifting the laser medium toward the shorter arm, if the laser is operated at $\delta/\delta_{\max} < 0.5$. For $\delta/\delta_{\max} > 0.5$ the laser medium has to be shifted toward the longer arm in order to gain higher modulation efficiency. The stability parameters for optimum amplitude modulation are the limits of the stability ranges at $\delta = -R_2$ and $\delta = -R_1$. Clearly, the hard-aperture KLM laser operates most efficiently near the limits of the stability ranges, and modulation efficiency has to be traded off against laser stability.

In conclusion, we have developed a self-consistent formalism for treating nonlinear resonators. This has allowed the derivation of analytical expressions for the passive amplitude modulation introduced by the combined effect of self-focusing and hard or soft aperture. For the gain aperture the negative influence of gain saturation on the working efficiency has been demonstrated, and design considerations have been given. The most important parameters for determining hard-aperture KLM have also been identified, and optimum design considerations have been discussed.

This research was supported by the Fond zur Förderung der wissenschaftlichen Forschung in Österreich grant P8391 and by the Österreichische Nationalbank Jubiläumsfondprojekt 4418. P. F. Curley acknowledges the support of the Royal Society.

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