# Periodic pulse evolution in solitary lasers

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Received September 22, 1992; revised manuscript received August 24, 1992

The periodic variation of femtosecond pulse characteristics within a mode-locked Ti:sapphire laser is investigated. By measurement of the output pulse characteristics at the dispersive and the nondispersive ends of a femtosecond Kerr-lens mode-locked laser cavity, it has been demonstrated that both the laser bandwidth and the pulse duration are functions of position within the laser cavity. A qualitative argument is also presented that predicts near-bandwidth-limited pulses at the cavity extremes, and this prediction is confirmed by experiment.

The generation of femtosecond pulse trains from Ti:sapphire lasers has now become a matter of course in ultrafast laser laboratories, with the most recent developments[1–4] in ultrashort pulse generation initiated by the discovery of self-mode locking.[5] In such dispersion-compensated lasers the final pulse duration is determined by the opposing effects of an intensity-dependent nonlinearity and intracavity dispersion. The mechanism for such pulse evolution in femtosecond lasers can therefore be described in terms of quasi-solitonlike pulse shaping that results from the competition between intracavity self-phase modulation within a Kerr medium and negative group-delay dispersion (GDD) introduced by an intracavity prism pair.[6] In contrast to soliton shaping, in quasi-solitonlike pulse shaping an additional passive amplitude modulation is required for the achievement of steady-state pulsed operation, and in Kerr-lens mode-locked (KLM) lasers this modulation is produced by Kerr lensing in the gain medium in combination with an intracavity aperture.

Previous analytical models for pulse evolution in these femtosecond lasers were based on the weak-pulse-shaping approximation,[7,8] in which all intracavity elements are assumed to affect the pulse characteristics simultaneously. However, as pulse energies and bandwidths increase, this assumption becomes less applicable, because the effects of nonlinearity and dispersion become nonnegligible as a result of discrete elements within the cavity.[9,10] Recent theoretical studies have taken this basic model a step further by using discrete operator equations to model the pulse propagation within the cavity, introducing the concept of the solitary laser.[9] In contrast to the soliton laser, in which the interplay between nonlinearity and dispersion occurs simultaneously within an optical fiber, the solitary model uses discrete nonlinear and dispersive elements combined with a weak stabilizing amplitude modulation arising from any gain or hard aperturing of the laser mode. In this paper we present a brief theoretical overview of the solitary laser and a direct comparison between the predicted laser behavior and experimental results for a KLM Ti:sapphire laser.

The electric field inside the mode-locked laser cavity may be written in the form $E(t) = v(t)\exp(i\omega t)$, where $v(t)$ is the complex pulse envelope and $t$ is measured in a frame of reference moving at the group velocity of the pulse. The pulse evolution at a cavity position $z$ can then be modeled by the expression $v_{n+1}(t, z) = \hat{T}(z)v_n(t, z)$, where $\hat{T}$ is the cavity transfer operator that may be written as a combination of the individual transfer operators of the discrete elements within one cavity round trip. For the dispersive cavity end $\hat{T}$ may be written as

$$\hat{T} = \exp(\hat{A})\exp(\hat{D})\exp(\hat{N})\exp(\hat{D}/2). \qquad (1)$$

The dispersion operator is given by $\hat{D} = i(D/2)(\partial^2/\partial t^2)$, where $\hat{A} = (g - l - \kappa|v|^2)/2$ is the operator describing the KLM amplitude modulation. The terms $g$ and $l$ are the saturated round-trip power gain and loss, respectively, and $\kappa$ is the effective cross section of the passive KLM modulator. The operator $\hat{N} = -i\phi|v|^2$ describes the Kerr nonlinearity,[9] the constant $\phi$ is the round-trip nonlinear phase shift (per watt), and $D(<0)$ is the net round-trip GDD. Steady-state mode-locking occurs if the condition $v_{n+1}(t, z) = \exp(i\psi)v_n(t, z)$ is satisfied, where $\psi$ is a constant phase shift and $n$ is the number of cavity round trips. It is instructive to introduce an $r$ parameter, defined as the ratio of the effective round-trip cavity length $L_r$ to the soliton period $L_s$, as a method of describing the level of discreteness of a solitary system compared with that of a purely soliton shaping mechanism. Hence $r = L_r/L_s = (1/2\pi)(\phi W)^2/(|D|)$, where $W$ is the intracavity pulse energy. With the approximation that the passive amplitude modulation is weak ($|\hat{A}v| \ll |\hat{N}v|$) and therefore insensitive to cavity position, the cavity transfer operator $\hat{T}(z)$ can be rewritten in the form[11]

$$\hat{T}(z) = \exp[\hat{A} + \hat{N}(r) + \hat{D}(r) + \hat{O}(r^2) + \hat{O}(r^3) + \ldots]. \qquad (2)$$

The $\hat{N}$ and $\hat{D}$ operators in the exponent describe a solitonlike system, and $\hat{O}(r^2)$ and $\hat{O}(r^3)$ are additional perturbation operators, which contain commutators and double commutators, respectively, of $\hat{N}$ and $\hat{D}$. The cavity-position-dependent coefficients of these operators can be determined by applying the Campbell–Baker–Hausdorff theorem.[12] At the two cavity ends it can be shown that the second-order perturbation term $\hat{O}(r^2)$ vanishes because of symmetry considerations.[11] The remaining third-order perturbation term $\hat{O}(r^3)$ is found to modify primarily the amplitude of the steady-state solution

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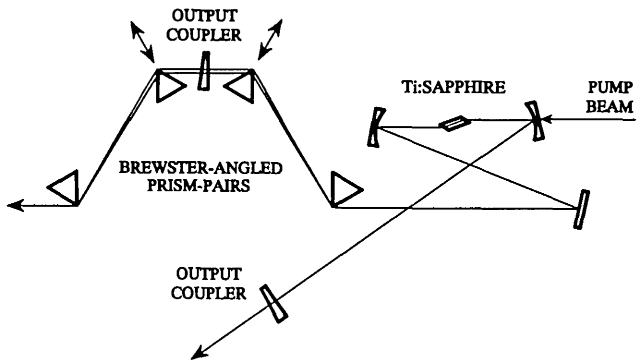


Fig. 1. Schematic of the KLM Ti:sapphire laser.

without introducing any additional chirp. Hence it follows that at the cavity extremes the phase of the mode-locked pulse is not modified (up to third order in $r$) and the pulse is thus approximately bandwidth limited. This qualitative argument is valid for $r$ values of the order of unity and can now be discussed in relation to the solitary pulse duration.

From a semiempirical solution it has been shown previously that for $r < 10$ the pulse duration within a solitary laser is given by[9]

$$\tau = \frac{3.53|D|}{\phi W} + \alpha\phi W, \tag{3}$$

where the variable $\alpha$ equals 0.10 and 0.25 at the dispersive (prism) and nondispersive cavity ends, respectively (Fig. 1). The quantitative implication of these derived results is that the pulse duration at the two cavity ends differs by $\Delta\tau = (0.25 - 0.10)\phi W = 0.15\phi W$. Hence, the conclusion that pulses at the cavity extremes are nearly bandwidth limited also implies that different spectral widths must also be present at the two cavity ends. Thus the characteristics of ultrashort pulses within a cavity of separate discrete nonlinear and dispersive elements are dependent on cavity position.

For pulse propagation from the nondispersive to the dispersive cavity end, the initial bandwidth-limited pulse experiences a chirp that is due to self-phase modulation in the Kerr medium and subsequent linear-chirp compensation after passage through the dispersive prism elements. This process is equivalent to conventional pulse compression, and hence the pulse is compressed relative to the initial duration. For the reverse case, propagation from the dispersive cavity end results in pulse broadening through the dispersive section, and the above results imply that bandwidth restriction in the Kerr medium must occur, producing the original bandwith-limited pulse. This is possible if the pulse incident upon the medium has a negative chirp. Using a synchronously pumped KLM Ti:sapphire laser, we investigate the experimental pulse parameters at both cavity ends of a solitary laser and compare the results with the theoretical considerations discussed above.

A schematic of the laser is shown in Fig. 1. The laser was constructed with an 8-mm-long Brewster-cut Ti:sapphire rod (Crystal Systems) as described in a previous study.[2] The laser cavity comprises two 100-mm-radius-of-curvature, high-reflector, single-stack mirrors, one high-reflector fold mirror, and two output couplers of 2.5% and 0.9% transmission. These low-transmission mirrors are selected to maximize the intracavity power and thus increase the intracavity pulse energy to a level at which the discrete solitary operation is enhanced. Two Brewster-angled prisms of F2 glass are positioned in one arm of the resonator, with an apex-to-apex separation of 40 cm. The laser is pumped by 5 W of 527.5-nm light from a mode-locked, frequency-doubled Nd:YLF pump laser (Quantronix 4216D). The Ti:sapphire laser is mode locked, and the passive intracavity amplitude modulation can be enhanced by use of a hard aperture in the nondispersive arm of the laser cavity. The mode locking is ini-

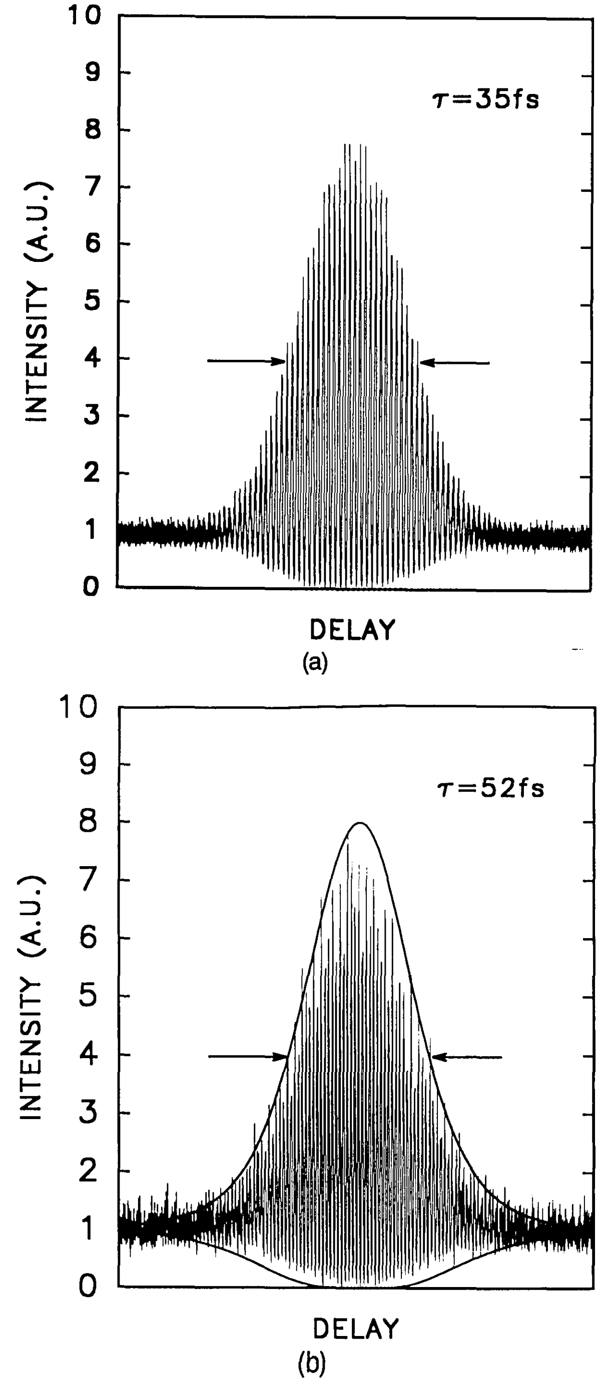


Fig. 2. Interferometric autocorrelations of the mode-locked pulses from (a) the dispersive cavity end and (b) the nondispersive end. Note the lack of any residual frequency chirp.

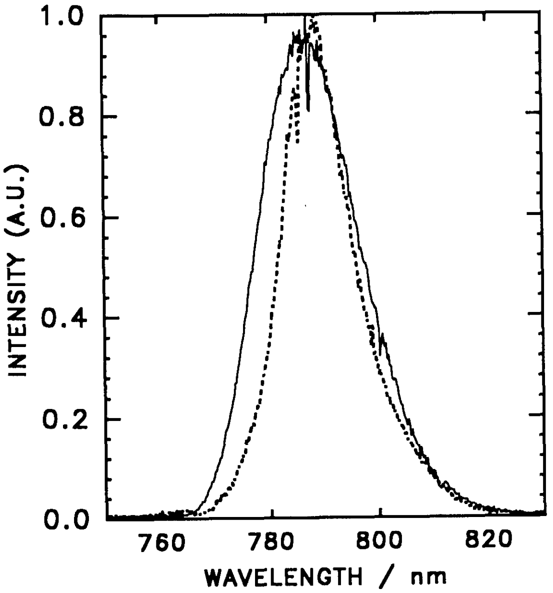

Fig. 3. Mode-locked spectra recorded at the dispersive cavity end (solid curve) and the nondispersive end (dashed curve).

tiated by matching the cavity length of the Ti:sapphire laser to that of the pump laser, which causes the KLM mechanism to start spontaneously from the synchronously pumped mode-locked state. The mode locking can tolerate cavity detuning away from the synchronously pumped state; however, the laser is not self-starting under such conditions. At the dispersive cavity end the output beam is spatially chirped as a result of passing through only a single prism pair. Hence a second pair of prisms is used externally to remove such chirp, and by the appropriate choice of the glass path within this prism section any additional chirp introduced by transmission through the output mirror substrate may also be compensated for.

The pulse spectra at the two cavity ends are recorded by use of an optical multichannel analyzer (EG&G Model 1460, ISA Instruments). The two output beams are directed into the analyzer, and the pulse spectra are recorded simultaneously by use of a two-dimensional multielement detector array (CCD, Model 1430-P). Autocorrelations of the pulses from both cavity ends are also recorded; however, these cannot be taken precisely simultaneously. A mirror is mounted onto a translation stage and positioned so that each output beam can be selected without the need for realignment of the autocorrelator. The fringe resolved autocorrelations are shown in Fig. 2 and indicate chirp-free pulses of $\tau \approx 35$ fs duration ($\mathrm{sech}^2$ shape assumed) at the dispersive output and $\tau \approx 52$ fs from the nondispersive cavity end. The signal-to-noise ratio is low for the latter measurement because of the low output from this arm of the resonator. The noise on the baseline of the autocorrelations originates from the pump laser modulator rf driver. Allowing for the dispersion of the mirror substrate, the calculated intracavity pulse duration at the output coupler of the nondispersive cavity end is $\tau \approx 48$ fs. This yields a measured difference in pulse duration of $\Delta\tau = 13$ fs between the two cavity ends. The pulse spectra from the two cavity ends are shown in Fig. 3, and the corresponding mode-locked bandwidths are 21.0 and 14.7 nm for the dispersive and nondispersive cavity ends, respectively. The corresponding time–bandwidth products $\tau\Delta\nu$ are 0.35 and 0.36, respectively.

A problem in applying Eq. (3) directly to femtosecond lasers is the difficulty of accurately measuring the net intracavity round-trip GDD. A new method has been proposed by Knox[13] for the determination of $D$; however, the reproducible wavelength tuning required was not possible with our present system. Nevertheless, with the measured prism parameters for our system, the net round-trip GDD can be estimated, yielding $D \approx -500\ \mathrm{fs}^2$. From the theoretical results discussed, it is also possible to estimate the magnitudes of both $D$ and $\Delta\tau$ from the measured mode-locked characteristics of our KLM system. For an average laser mode spot size in the Kerr medium of 28 $\mu$m, $\phi = 1.9 \times 10^{-6}\ \mathrm{W}^{-1}$, and, for intracavity pulse energies of $eW = 46$ nJ, substitution into Eq. (3) yields the result $D \approx -650\ \mathrm{fs}^2$, which is in reasonable agreement with the calculated net GDD based on the measured glass paths of the beams within the prisms. Similarly, we can obtain a predicted result for $\Delta\tau = 13.2$ fs, a result calculated independent of the dispersion $D$, which is in good agreement with the experimentally determined value.

In summary, we have demonstrated the operation of a solitary laser and confirmed that mode-locked pulse characteristics such as duration and bandwidth can vary dramatically as a function of cavity position. In such solitary lasers the discrete pulse-shaping effects can be strong enough to produce considerable modification to the pulse over a single cavity round trip. As a consequence, the weak-pulse-shaping approximation (i.e., $r \ll 1$) cannot be assumed to be valid for femtosecond solitary lasers, and the conventional description for mode locking in terms of phase-locked intracavity modes cannot be generally applied to such systems.

## ACKNOWLEDGMENTS

Helpful conversations with A. J. Schmidt are gratefully acknowledged. P. F. Curley acknowledges the support of the Royal Society, London, in the form of a European Exchange Fellowship. This work was supported by Fonds zur Förderung der Wissenschaftlichen Forschung in Österreich grant P8566-TEC.

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