Passive mode locking in standing-wave laser resonators

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In standing-wave laser oscillators the field energy density periodically varies along the resonator axis and creates a complex refractive-index grating in the gain medium. This grating couples the originally independent counterpropagating waves of the individual axial cavity modes. The coupling induces a mode frequency shift that is a nonlinear function of the unperturbed mode frequency. The uneven shifts can give rise to a substantial broadening of the beat-note linewidth of the multi-axial-mode free-running laser and to a corresponding increase in the threshold intracavity power for self-starting passive mode locking. The theoretical results are in good qualitative and quantitative agreement with previously reported experimental observations and measurements.

Passive mode locking of cw lasers has been a reliable technique for the generation of ultrashort optical pulses. Whereas a suitable combination of a slow saturable absorber and saturable gain can generally ensure spontaneous start-up in dye lasers, solid-state lasers mode locked by fast saturable absorberlike nonlinearities often require very high intracavity powers to initiate self-starting.[1,2] Improvement of the self-starting performance of femtosecond solid-state lasers calls for a deeper understanding of the basic physical processes influencing the transient buildup of mode locking. In this Letter we propose a mechanism that might ultimately be responsible for determining the minimum intracavity power needed for self-starting passive mode locking in standing-wave laser cavities.

It has been known since the early years of laser mode locking that the passively mode-locked pulse evolves from an initial intensity fluctuation formed by mode beating in multi-axial-mode laser oscillators.[3] Recently it was pointed out[4] that in a cw-pumped free-running laser random mode beating fluctuations decay and emerge within a characteristic time that can be referred to as the lifetime of the fluctuations. For mode locking to prevail, the excess round-trip gain experienced by the initial fluctuation must be large enough to complete the mode locking process within the lifetime of the fluctuation. This lifetime is related to the linewidth of the first beat note in the rf power spectrum of the free-running laser,[4] and thus it is easily accessible experimentally. The finite beat-note linewidth was attributed to two distinct different physical effects: (i) the finite mutual coherence time of the longitudinal modes and (ii) mode pulling, which is nonlinear in the mode frequency. A primary consequence of the latter effect is uneven frequency spacings between adjacent longitudinal modes. Since the mode spacing is given by the ratio of the phase velocity of light to the round-trip cavity length, the effect of uneven mode spacings can be described in terms of a phase velocity that is slightly different for different axial modes. Clearly, a frequency-dependent phase velocity leads to a decay of the initial pulselike fluctuation just as the finite mutual coherence of the axial modes.

Recently Haus and Ippen[5] suggested that spurious intracavity reflections can lead to nonlinear mode pulling and showed theoretically their adverse effects on mode locking. However, subsequent experiments[2] indicated that after elimination of all identifiable spurious intracavity and extracavity reflections, the beat-note linewidths of optimized systems exhibit a strong dependence on both the intracavity power and the total internal cavity losses. This behavior suggests that the beat-note linewidth is ultimately affected by optical nonlinearities in the cavity. One nonlinear element present in every laser is the gain medium. In a two-mirror resonator the standing-wave pattern of the electric field induces a spatial modulation[6] of the complex refractive index in the gain medium. In this Letter we show that the grating etched in the gain medium can lead to a significant broadening of the beat-note linewidth in standing-wave resonators, which may severely affect the self-starting passive mode-locking performances of these systems.

Consider the laser oscillator illustrated in Fig. 1. We assume a spatial modulation of the complex refractive index $n_c(z)$ in the gain medium of the form

$$n_c(z) = n + j\frac{c}{\omega}\alpha + \left(n_1 + j\frac{c}{\omega}\alpha_1\right)\cos(2\beta_0 z + \varphi), \tag{1}$$

where n and α are the refractive index and the gain coefficient, respectively, n_1 and α_1 are the amplitudes of the corresponding spatial modulation, ω denotes the frequency of the selected mode (the mode index has been dropped), $\beta_0 = n\omega_0/c$ is the wave number of the standing-wave grating, and φ is a constant

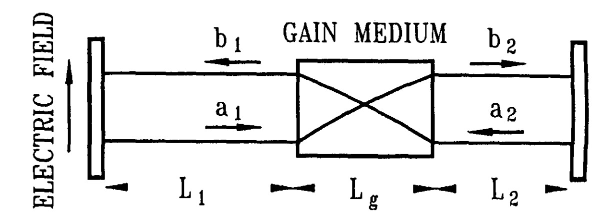

Fig. 1. Circulating radiation in a linear laser resonator under steady-state conditions. The intracavity losses have been incorporated into the mirror reflectivities.

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phase shift. The sign of the imaginary part of n_c is chosen such that there is gain for $\alpha > 0$. Further, it is assumed that $n_1 \ll n$, $\alpha_1 \ll \alpha \ll \beta_0$, and $|\omega - \omega_0|/\omega_0 \ll 1$. These approximations enable us to take advantage of Kogelnik and Shank's theory of distributed-feedback lasers.[7] Following the coupled-wave analysis presented in Ref. 7, we can express the amplitudes of the waves exiting the gain medium (b_1 and b_2 in Fig. 1) as a linear combination of those of the incoming waves (a_1 and a_2) of the form

$$b_1 = \exp[(g_s/4) - j\phi_g](Ka_1 + a_2), \tag{2a}$$

$$b_2 = \exp[(g_s/4) - j\phi_g](a_1 + Ka_2), \tag{2b}$$

where

$$K = -j\epsilon L_g \frac{\sin\Delta}{\Delta}, \tag{3a}$$

$$\epsilon = \frac{1}{2}(\beta_0 n_1 + j\alpha_1). \tag{3b}$$

Here $g_s = 4\alpha L_g$ is the round-trip saturated power gain, $\phi_g = (\omega/c)nL_g$ stands for the single-pass phase delay of the gain medium, and $\Delta = (\omega - \omega_0)nL_g/c$ is a normalized frequency parameter. In the derivation of Eqs. (2) and (3) a weak coupling ($|\epsilon|L_g \ll 1$) and a low-loss laser cavity ($g_s \ll 1$) were assumed. Feedback from the resonator mirrors yields the additional constraints for the steady-state field amplitudes:

$$a_i = \exp[-(l_i/2) - j\phi_i]b_i, \qquad i = 1, 2. \tag{4}$$

Here l_i, $\phi_i = 2(\omega/c)L_i$, and L_i represent the round-trip internal cavity loss, phase delay, and length of the corresponding cavity arm, respectively.

For steady-state oscillation, the determinant of the system, Eqs. (2) and (4), must equal zero. Following Haus and Ippen,[5] we first solve the determinantal equation of the unperturbed cavity and subsequently derive an equation for the change of the saturated gain δg_s and resonance frequency $\delta\omega$ to first order in ϵL_g, δg_s, and $\delta\omega$. The result is

$$\frac{1}{4}\delta g_s + j\frac{L_r}{c}\delta\omega = \epsilon L_g \frac{\sin\Delta}{\Delta}\left[\frac{g_s}{4}\sin(\phi_1 + \phi_g) - j\cos(\phi_1 + \phi_g)\right], \tag{5}$$

where $L_r = L_1 + L_2 + nL_g$ is the effective resonator length. We arrived at Eq. (5) by assuming an asymmetric distribution of cavity losses $l_1 \approx l$ and $l_2 \approx 0$, which is typical of practical systems. Equation (5) accounts for a slight change of the steady-state gain and mode frequency, whose unperturbed values are $g_s = l$ and $\omega = 2\pi m_k \Delta\nu_0$, respectively. Here m_k is the number of half-wavelengths of the kth mode in the cavity and $\Delta\nu_0$ is the mode spacing in the absence of frequency-pulling effects. The nonlinear relationship between $\delta\omega$ and the mode index k gives rise to a change $\Delta\omega_k = \delta\omega_{k+1} - \delta\omega_k$ in the mode beating frequencies that is different for different pairs of adjacent axial modes. This in turn results in a broadening of the spectral line observed at the frequency $\Delta\nu_0$ (first beat note) in the power spectrum of the free-running laser. For an infinite mode coherence time the linewidth of the first beat note $\Delta\nu_{3\mathrm{dB}}$ (Ref. 4) can be estimated by calculating the root mean square of $\Delta\nu_k = \Delta\omega_k/2\pi$ for the free-running axial modes.

In general, no closed-form expression can be obtained for $\langle\Delta\nu_k^2\rangle^{1/2}$. Hence we restrict our discussion to a specific case in order to present some representative quantitative results. The assumptions used are (a) that the laser medium is much shorter than the resonator length and is positioned approximately in the middle of the cavity, (b) that the number of free-running modes N is much higher than 1, and (c) that N does not significantly exceed L_r/L_g. From assumptions (a) and (b) it follows that $\phi_1 + \phi_g$ is randomly distributed between 0 and 2π for the oscillating modes, implying that $\langle\sin^2(\phi_1 + \phi_g)\rangle^{1/2} = \langle\cos^2(\phi_1 + \phi_g)\rangle^{1/2} = 1/\sqrt{2}$ and $\langle\sin(\phi_1 + \phi_g)\cos(\phi_1 + \phi_g)\rangle^{1/2} = 0$. Assumption (c) allows $\sin\Delta/\Delta$ to be replaced by unity in Eq. (5). With these approximations, Eqs. (3b) and (5) yield

$$\Delta\nu_{3\mathrm{dB}} \approx 2\langle\Delta\nu_k^2\rangle^{1/2} = \sqrt{\Delta\nu_\alpha^2 + \Delta\nu_n^2}, \tag{6}$$

where

$$\Delta\nu_\alpha = \frac{g_s}{2\pi\sqrt{2}}|\alpha_1|L_g\Delta\nu_0 \tag{7}$$

is the broadening induced directly by the periodic gain modulation and

$$\Delta\nu_n = \frac{\sqrt{2}\beta_0}{\pi}|n_1|L_g\Delta\nu_0 \tag{8}$$

is a contribution from the refractive-index grating. The self-starting threshold in the absence of beat-note line broadening effects other than the complex index grating in the gain medium can now be obtained by substituting relation (6) into the general formula[4]

$$\kappa P > \frac{\pi}{\ln N}\frac{\Delta\nu_{3\mathrm{dB}}}{\Delta\nu_0}, \tag{9}$$

where κ is a characteristic of the fast saturable absorberlike nonlinearity[2] used for passive mode locking and P is the circulating average intracavity power in the free-running laser.

We now consider the origin of the complex index modulation given by Eq. (1). The steady-state field energy density in the gain medium is given by

$$\rho(z) = \rho + \rho_1(z) = \sum_{k=-(N-1)/2}^{(N-1)/2}\rho_k - \sum_{k=-(N-1)/2}^{(N-1)/2}\rho_k\cos\frac{2\pi m_k}{L_r}(nz - L_1), \tag{10}$$

where ρ_k is the spatially averaged energy density of the kth mode and $z = 0$ at the left-hand side of the laser medium (Fig. 1). The first sum in Eq. (10) gives the total average energy density ρ. The assumption of $N \gg 1$ implies that $\rho_1(z) \ll \rho$, and

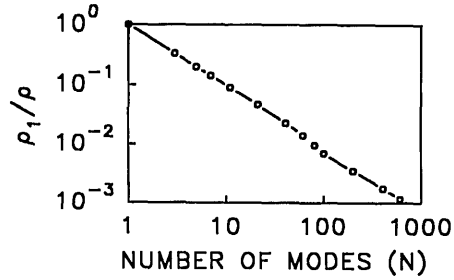

Fig. 2. Effective modulation amplitude (ρ_1) of the energy density relative to the average energy density (ρ) in the gain medium as a function of the number of oscillating modes (N) in a resonator with $L_r = 1$ m, $m_0 = 2 \times 10^6$ ($\lambda_0 = 1\ \mu$m), $L_1 = L_2$, $nL_g = 10$ mm, and $\rho_k = \rho/N$. The effective modulation amplitude is defined by $\rho_1 = \sqrt{2}\int_0^{L_g} \rho_1{}^2(z)\mathrm{d}z$, where the factor $\sqrt{2}$ ensures that $\rho_1/\rho = 1$ for $N = 1$.

thus the spatially varying part of the steady-state (saturated) gain coefficient can be written as

$$\alpha(z) - \frac{g_s}{4L_g} \approx \alpha_1 \cos\left(2\frac{\omega_0}{c} nz + \varphi\right), \qquad (11a)$$

$$\alpha_1 = \frac{g_s}{2L_g}\frac{P}{P_s}\left(1 + \frac{2P}{P_s}\right)^{-1}\frac{\rho_1}{\rho}. \qquad (11b)$$

Here P_s is the saturation power of the gain medium, ω_0 represents the mode frequency for $k = 0$, and ρ_1 is the effective modulation amplitude of $\rho_1(z)$, as defined in the caption of Fig. 2. Approximation (11a) is reasonable as long as assumption (c) is justified. The explicit dependence of α_1 on the intracavity power vanishes for $P/P_s \gg 1$. Figure 2 plots ρ_1/ρ versus N for a representative case characterized in the figure caption.

The real part n_1 of the complex index modulation can now be calculated by use of the Kramers–Kronig theorem. For homogeneously broadened lasers, which have a free-running oscillation bandwidth much smaller than the fluorescence linewidth, relating n_1 to α_1 (Ref. 8) yields $\Delta\nu_n \ll \Delta\nu_\alpha$. With $\Delta\nu_0 = 100$ MHz, $g_s = 0.2$, $P/P_s = 2$, and $N \approx 10^2$, which implies that $\rho_1/\rho \approx 10^{-2}$ (see Fig. 2), substitution of Eq. (11b) into Eq. (7) yields $\Delta\nu_\alpha \approx 1$ kHz for a typical cw solid-state laser. Considering the fact that $\Delta\nu_\alpha$ is expected to be broader for an asymmetrically located gain medium because of stronger spatial hole burning (larger ρ_1/ρ), this prediction is in reasonable agreement with recent experimental results.[2] An important implication of the theory is the explicit quadratic dependence of $\Delta\nu_\alpha$ on g_s, i.e., on the net intracavity loss.[9] The rapid increase in the beat-note linewidth with increasing cavity loss has been experimentally verified in Nd:YLF, Nd:glass, and Ti:sapphire lasers[2] and more recently also in a Nd:fiber laser.[10]

Beyond this quantitative comparison, some qualitative experimental observations can also be interpreted in terms of our theory. Since it takes a finite time for the population grating to build up, a fast enough displacement of the standing-wave pattern of the field energy density in the gain medium can lead to a reduction of α_1. Hence a sudden perturbation[11] or periodic variation[12] of the cavity length implies not only increased mode-beating fluctuations but also a partially erased population grating in the gain medium. Previous observations[13] in an actively mode-locked system support this hypothesis. One can eliminate the complex index grating in the gain medium most efficiently by constructing a ring cavity in which only one propagation direction is allowed. Recently it was demonstrated experimentally that unidirectional ring cavities improve the start-up performance of both actively[13] and passively[14] mode-locked lasers.

These findings and the reasonable quantitative predictions of our theory suggest that the beat-note line broadening originating from spatial hole burning in the gain medium is likely to be the dominant effect opposing self-starting passive mode locking in practical standing-wave laser oscillators. It is hoped that further research on the ideas presented in this Letter will pave the way for improved passive mode-locking performance of these systems.

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