Measurement of interferometric autocorrelations: comment

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Conventional Michelson interferometers that are widely used for autocorrelation of ultrashort optical pulses are incapable of producing two identical replicas of an optical pulse, hence yielding a correct autocorrelation signal. The problem is acute when one records the interferometric autocorrelation of pulses consisting of just a few optical cycles. With the use of sub-10-fs pulses we demonstrate asymmetric autocorrelation traces recorded with a conventional Michelson interferometer, refer to previously reported results that exhibit the same deficiency, and elucidate the origin of the problem. A simple modification of the conventional interferometer configuration yields the correct fringe-resolved autocorrelation trace. © 1997 Optical Society of America

Nonlinear autocorrelation with optical second-harmonic generation has been the standard technique for measuring the duration of ultrashort optical pulses.[1] The collinear interferometric, fringe-resolved autocorrelation (FRAC) technique first demonstrated by Diels *et al.*[2] has proved superior to its noncollinear background-free counterpart in several respects. One of its major advantages is the comparatively high sensitivity of the calculated FRAC envelopes to the pulse parameters. For the experimentalist, other important benefits include the direct and accurate self-calibration provided by the interferometric fringes, and the possibility of making sure that the autocorrelator is properly aligned by checking the contrast ratio. FRAC measurements also allow a full amplitude and phase retrieval by means of iterative procedures.[3,4]

A practical implementation of the FRAC technique is fairly straightforward, nevertheless special care has to be taken in splitting the incident beam, if the correct theoretical FRAC trace is to be measured. To our knowledge this issue has not received attention before now, and here we show that a conventional FRAC setup does not allow recording of the correct FRAC trace. Although the deviation of the measured correlation signal from the theoretical FRAC function is scarcely perceivable for moderately short pulses, significant deviations arise when the pulse duration becomes comparable to the optical cycle of the carrier.

Figure 1 illustrates the schematic of a collinear autocorrelator as it is widely used for FRAC measurements. For measuring the duration of optical pulses in the 10-fs regime, an antireflection-coated duplicate of the beam splitter substrate has to be inserted (at 45°) between mirror M1 and the beam splitter for balancing the dispersion in the two arms of the interferometer. In our experimental setup we used a quarterwave dielectric 50/50 beam splitter for *s*-polarized light deposited onto a 0.3-mm-thick fused-silica substrate, an identical antireflection-coated compensation plate, and an off-axis parabola for focusing the output of the interferometer onto a 25-μm-thick β-barium borate frequency-doubling crystal. This experimental arrangement was used to measure the FRAC trace of the sub-10-fs output of a self-mode-locked, mirror-dispersion-controlled Ti:sapphire laser.[5] The result is shown in Fig. 2. The correlation trace reveals a minimum close to its center of gravity, hence it cannot represent a FRAC trace. Both the interferometric correlation trace of the 6-fs pulses generated at AT&T Bell Laboratories in 1987 (Ref. 6) as well as a series of measurements on ultrashort-pulse Ti:sapphire lasers published by our group in recent years[5,7,8] suffer from this shortcoming. In what follows, we shed light on the origin of this problem, investigate the implications of previous incorrect FRAC measurements, and demonstrate how a simple

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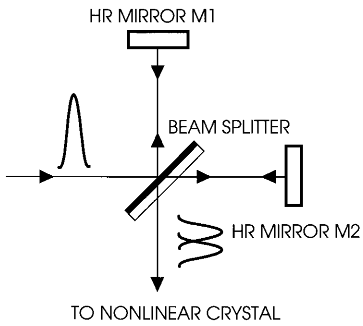

Fig. 1. Setup of a standard collinear autocorrelation.

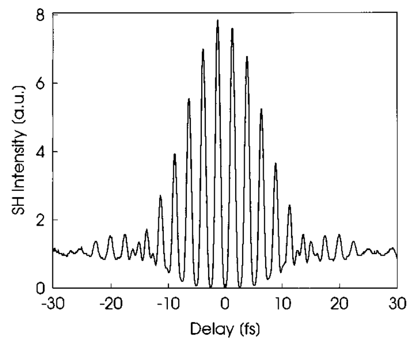

Fig. 2. FRAC of a sub-10-fs pulse measured with a standard autocorrelator. Note the minimum at zero delay.

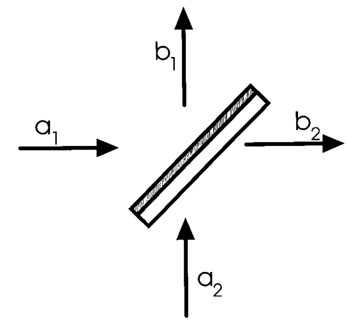

Fig. 3. Complex amplitudes a_j of incident and b_i reflected waves on a beam splitter.

modification of the standard FRAC configuration yields the correct autocorrelation trace.

We start by considering the response of a Michelson interferometer to an incident wave packet. The action of a beam splitter on incident monochromatic waves can be described in terms of its scattering matrix[9]:

$$b_1(\omega) = S_{11}a_1(\omega) + S_{12}a_2(\omega), \tag{1a}$$

$$b_2(\omega) = S_{21}a_1(\omega) + S_{22}a_2(\omega), \tag{1b}$$

where the components of the scattering matrix S_{ij} relate the complex amplitudes a_j of incident waves to those b_i of the reflected waves (Fig. 3). After Fourier decomposition of the incident pulse, this formalism can be used to calculate the response of the Michelson interferometer shown in Fig. 1. For the Fourier spectrum of the output straightforward algebra yields

$$\begin{aligned} E_{\text{out}}(\omega) = \{&S_{11}(\omega)\exp[i\phi_1(\omega)] \\ &+ S_{22}(\omega)\exp[i\phi_2(\omega)]\}S_{12}(\omega)E_{\text{in}}(\omega), \end{aligned} \tag{2}$$

where $E_{\text{in}}(\omega)$ represents the Fourier components of the pulse impinging on the interferometer and $\phi_i(\omega) = 2(\omega/c)l_i$ ($i = 1, 2$), where l_1 and l_2 are the optical lengths of the interferometer arms. In the derivation of Eq. (2) we assumed that the beam splitter is a reciprocal optical element and hence $S_{21} = S_{12}$.[9] The phase factors in Eq. (2) imply a distortion-free delay of the two replicas of the incident pulse in the interferometer arms (we have neglected dispersion introduced by the beam splitter substrate and compensation plate).

The phase of the complex reflection coefficients S_{ii} can be expanded in a power series about the center frequency of the beam splitter:

$$\begin{aligned} \varphi_{ii}(\omega) &\approx \varphi_{ii}(\omega_0) + \varphi_{ii}'(\omega_0)(\omega - \omega_0) \\ &= \alpha_{ii} + \tau_{ii}\omega, \quad i = 1, 2. \end{aligned} \tag{3}$$

This linear approximation provides a good description of the frequency dependence of a dielectric beam splitter for the wavelength range over which the splitting ratio is ≈50/50 (for s-polarized light this bandwidth exceeds 250 nm centered around 800 nm with TiO_2 and SiO_2 as the high- and low-index materials). Because of the inherent structural asymmetry (air–coating substrate) of dielectric (or metallic) beam splitters, the coefficients α_{ii} and τ_{ii} are generally different for the air ($i = 1$) and substrate sides of the coating. $\tau_{11} \neq \tau_{22}$ merely implies that zero relative delay of the two replicas of the pulse can be realized with a properly biased interferometer, $(l_2 - l_1)/c = \tau_{11} - \tau_{22}$, rather than at $l_2 = l_1$. Unfortunately, a difference between α_{11} and α_{22} cannot be compensated in such a simple way. The physical meaning of $\alpha_{ii} \neq 0$ is a shift of the carrier wave with respect to the pulse envelope by α_{ii}. Hence, $\alpha_{22} \neq \alpha_{11}$ implies different shifts of the carrier relative to the envelope in the two replicas of the pulse. This, in turn, leads to a fringe-resolved correlation signal $S(\tau)$ that cannot meet the condition $S(0)/S(\infty) = 8$ and usually does not meet the condition $S(-\tau) = S(\tau)$ either, where τ is the difference between the delays of the two interferometer arms.

In our case $\varphi_{11} = \pi$, $\varphi_{22} = 0$, yielding $\alpha_{11} - \alpha_{22} \approx \pi$. This gives rise to $S(0) \approx 0$ in the measured fringe-resolved correlation trace, as shown in Fig. 2. As mentioned above, similar deviations from the theoretical FRAC trace were observed in previous experiments.[5–8] In this context the question arises: does the asymmetric beam splitter lead to incorrectly evaluated pulse durations in these previous experiments. To answer this question, we calculated the fringe-resolved correlation traces of an 8-fs Gaussian-shaped pulse for $|\alpha_{11} - \alpha_{22}| = \pi$ and $\alpha_{11} - \alpha_{22} = 0$ (correct FRAC trace). Figure 4 reveals that both the correct and the phase-shifted FRAC trace fit precisely between the same envelopes. As a consequence, if the pulse duration is evaluated from the width of the envelope of the fringe-resolved correlation trace, the phase asymmetry discussed above does not lead to an error in the measurement of the pulse duration.

For pulses consisting of just a few optical cycles, however, the envelope of the FRAC trace can be only uniquely determined for analytic (e.g., Gaussian or secant hyperbolic) pulse shapes. As a consequence, a more sophisticated pulse analysis that does not assume an analytic pulse shape (either because this is not justified or because small deviations from an

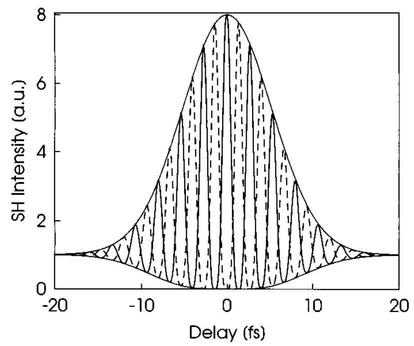

Fig. 4. Calculated FRAC traces of an 8-fs Gaussian pulse. The solid and dashed curves correspond to a phase difference of 0 and π, respectively. Both curves fit well to the same envelope.

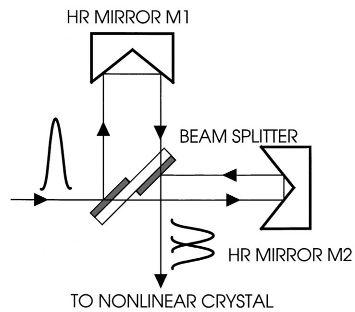

Fig. 5. Setup of a symmetric collinear autocorrelator.

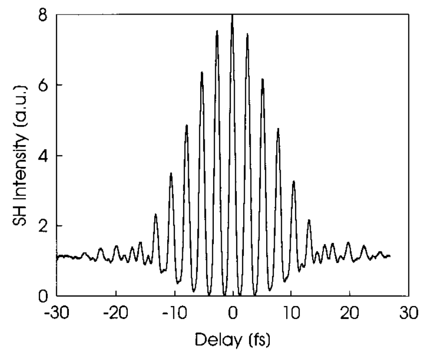

Fig. 6. FRAC of a sub-10-fs pulse measured with a symmetric autocorrelator. Note the maximum at zero delay, in comparison with Fig. 2.

analytic pulse form are to be determined) can be based only on a comparison of the full measured and calculated FRAC trace. Of course, this is possible only if the correct FRAC characterized by $S(0)/S(\infty) = 8$ can be measured.

It is clear from the above discussion that one can perform correct FRAC measurements only by using a symmetric beam-splitting system. This can be realized by the configuration sketched in Fig. 5. This symmetric Michelson interferometer employs two (identical) beam splitters that are set up in such a way that both replicas of the pulse that are recombined in the output arm of the interferometer are reflected off the same (air) side of the beam splitter. As a consequence, the transmitted waveform is the sum of two identical waveforms delayed with respect to one another:

$$E_{\rm out}(\omega) = \{\exp[i\phi_1(\omega)] + \exp[i\phi_2(\omega)]\} \times S_{11}(\omega)S_{12}(\omega)E_{\rm in}(\omega), \tag{4}$$

yielding the correct FRAC trace regardless of the specific properties of the beam splitter. Note that this configuration also reduces the amount of material dispersion in the beam path by a factor of 3 compared with the standard system in Fig. 1, as pointed out recently by Barty *et al.*[10] The suitability of this setup for correct FRAC measurements is clearly demonstrated by Fig. 6, which shows the FRAC trace of the same laser output as that used for the measurements employing the setup shown in Fig. 1.

In conclusion, special care has to be taken in the implementation of the fringe-resolved autocorrelation technique if extremely short pulses are to be measured. Only the use of a perfectly symmetric interferometric configuration allows the measurement of the exact FRAC trace, which is important for a precise assumption-free characterization of optical pulses in the 10-fs domain.

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