Carrier envelope phase noise in stabilized amplifier systems

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At present most laser systems for generating phase-stabilized high-energy pulses are chirped pulse amplifier systems that involve the selection and subsequent amplification of pulses from a phase-stabilized seed oscillator. We investigate the effect of the picking process on the carrier envelope phase stability and how the phase noise of the picked pulse sequence can be estimated from the phase noise properties of the seed oscillator. All noise components from the original pulse train above the picking frequency are aliased into the picked pulse train and therefore cannot be neglected.

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Ultrashort high-energy pulses with a stabilized carrier envelope phase[1] (CEP, the phase offset between pulse envelope and electric field maximum) play an increasing role in ultrafast physics and are particularly important for generation and observation of phenomena on an attosecond time scale.[2–4] Typical laser systems generating such pulses consist of a femtosecond laser oscillator that operates with a repetition rate ω_r of the order of 100 MHz, a pulse picking system (e.g., a Pockels cell) that selects every Pth pulse from that seed oscillator, and an amplifier system that operates in a range from 1 Hz to 100 kHz repetition rate into which these picked pulses are fed.[5]

If the carrier envelope offset frequency ω_{CE}, i.e., the time derivative of the CEP, is controlled appropriately, it can be arranged that every Rth pulse has the same CEP, and if only such pulses are selected for amplification (pulse picking) an amplified pulse train with constant CEP may be derived.[2] As the phase-locked loop used for controlling and stabilizing the CEP of the seed oscillator will always have a finite bandwidth and the system used to detect the CEP will always be subject to noise, the CEP will also be noisy even if the electronic circuits used for phase stabilization work perfectly. Pulse picking can be understood as a sampling of the seed oscillator CEP and is therefore subject to what is called aliasing in signal processing.

In this Letter we present how the CEP noise properties of the picked pulse sequence relate to the phase noise properties of the seed oscillator and how an estimate of the power spectral density (PSD) of CEP fluctuations of the picked pulse train can be obtained. Integrating this function within appropriate bounds yields the root mean square (rms) phase error for a given integration time and bandwidth, which is the relevant number for judging the quality of the CEP stabilization when one is considering experiments that are sensitive to the CEP.

Typically, in ultrafast applications one deals with a pulse train, whose electric field can be written as

$$E(t) = \sum_{n=-\infty}^{\infty} A(t - nT_r)\exp(i\phi_n) + \text{c.c.}, \tag{1}$$

where $T_r = 2\pi/\omega_r$ is the repetition period of the pulse train and ϕ_n is the CEP of the nth pulse. Function $A(t)$ is the complex electric field amplitude of a single pulse that vanishes for $|t| > T_r/2$ (or falls off sufficiently fast to make the above sum converge). ϕ_n forms a discrete time series, as the CEP has no meaning between pulses [Fig. 1(a)].

As the pulse-to-pulse CEP change is given by[6] $\Delta\phi = \phi_{n+1} - \phi_n = 2\pi\omega_{\mathrm{CE}}/\omega_r$, it is convenient to lock the CE frequency to an integer fraction R (here uppercase letters denote fixed integers and lowercase letters denote running integers) of the repetition frequency so that

$$\phi_n = \phi_0 + n2\pi/R + \psi_n, \tag{2}$$

where we have added ψ_n that represents the deviations from the set point due to imperfections in phase lock. The phase-locking electronics ensures that ψ_n is zero on average, and therefore the electric field reproduces itself after R pulses on average. If a pulse-picking system is employed to select every Pth pulse with phase ψ_{Pn} and P being an integer multiple of R, the CEP of the picked pulse train is given by

$$\phi_{Pn} = \phi_0 + Pn2\pi/R + \psi_{Pn} = \phi_0 + \psi_{Pn}. \tag{3}$$

This means that on average only identical pulses with the same CEP (ϕ_0) are amplified but the unavoidable phase fluctuations ψ_{Pn} will still be visible in the picked pulse sequence. The relation between ψ_n and ϕ_{Pn} is obvious only for the case where ψ_n is known completely. Typical figures to quantify noise processes, where ψ_n is a statistical quantity, are the rms value of ψ_n within some given bandwidth, the average PSD, and the Allan variance.[7] Here we shall concentrate on the PSD.

Let us assume for now that the CEP error of the seed oscillator for N pulses ψ_n, $n = 0 \ldots N-1$, has been

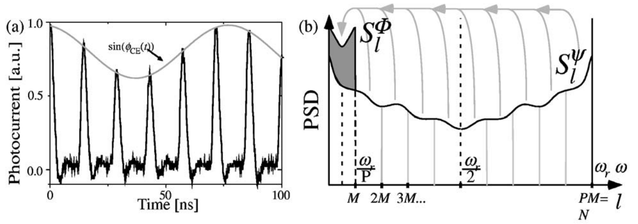

Fig. 1. (a) Pulse train as observed after a PPLN CEP detector with a bandwidth larger than ω_r.[9] The interferometer imposes a modulation with the frequency ω_{CE} on the original pulse train that indicates the CEP. (b) Different portions of the original PSD are aliased into the PSD of the picked pulse train and add there incoherently. Index l (see text) corresponds to the frequencies as given in the graph.

measured (exactly). In Fourier space this can be represented using a discrete Fourier transform:

$$\psi_n = \frac{1}{N}\sum_{k=0}^{N-1} \hat{\psi}_k \exp(i2\pi nk/N), \tag{4}$$

where $\hat{\psi}_k$ is the Fourier component corresponding to frequency $\omega_k=2\pi k/NT_r$ and the frequency resolution is given by the inverse of the observation time, $\Delta\omega=2\pi/NT_r$. The highest frequency appearing here is half the repetition frequency of the seed oscillator, which is the Nyquist frequency $\omega_{\mathrm{Nyq}}=2\pi N/2NT_r=\omega_r/2$.[8] The components of the Fourier transform above $k=N/2$ correspond to the negative frequencies in continuous Fourier transform. This fact implies that to measure ψ_n completely a measurement system with a bandwidth equal to or larger than this Nyquist frequency $\omega_r/2$ is necessary. The so-called two-sided PSD S_k^{ψ} of ψ_{n}, whose integral is (via Parseval's theorem) the mean square value of ψ_n, is defined as

$$S_k^{\psi} := \frac{T_r}{N}|\hat{\psi}_k|^2. \tag{5}$$

The CEP error of the picked pulse train can be written in terms of the Fourier components of the original pulse train:

$$\Phi_n := \psi_{Pn} = \frac{1}{N}\sum_{j=0}^{P-1}\sum_{l=0}^{M-1} \hat{\psi}_{jM+l} \exp[i2\pi Pn(jM+l)/N], \tag{6}$$

which is just a rearrangement of the sum in Eq. (4) that requires $N=PM$ (M being the total number of picked pulses), which can always be fulfilled by adjusting the total observation time NT_r. Then $\exp[i2\pi Pn(jM+l)/N]=\exp(i2\pi nl/M)$ and Eq. (6) can be rewritten as

$$\Phi_n = \frac{1}{M}\sum_{l=0}^{M-1}\left(\frac{1}{P}\sum_{j=0}^{P-1} \hat{\psi}_{jM+l}\right)\exp(i2\pi nl/M). \tag{7}$$

Comparing Eq. (7) with Eq. (4), the term in parentheses is identified as the Fourier components $\hat{\Phi}_l$ of Φ_n where the index n, which now runs from $n=0\ldots M-1$, numbers the picked pulses. In analogy with Eq. (5) the PSD of the picked sequence in Eq. (7) is given by

$$S_l^{\Phi} = \frac{PT_r}{M}|\hat{\Phi}_l|^2 = \frac{T_r}{N}\left|\sum_{j=0}^{P-1} \hat{\psi}_{jM+l}\right|^2. \tag{8}$$

As the range of index l is reduced, the highest-frequency component that appears is now $2\pi M/2NT_r=\omega_r/2P$, which is the Nyquist frequency of the picked pulse train. However, the PSD in Eq. (8) still contains contributions of all higher frequencies of the spectrum of the original pulse train aliased to frequencies below $\omega_r/2P$. Aliasing always happens if a function that is sampled at discrete times contains Fourier components higher than the Nyquist frequency determined by the sampling rate.

All the statements above are true for any function ψ_n. Taking the average over a typical ensemble of such functions from a stationary noise process, the average PSD of the picked pulse train can be related to the average PSD of the original pulse train by virtue of

$$\langle S_l^{\Phi}\rangle = \left\langle \frac{T_r}{N}\left|\sum_{j=0}^{P-1}\hat{\psi}_{jM+l}\right|^2\right\rangle = \frac{T_r}{N}\sum_{j=0}^{P-1}\langle|\hat{\psi}_{jM+l}|^2\rangle = \sum_{j=0}^{P-1}\langle S_{jM+l}^{\psi}\rangle, \tag{9}$$

where the second equally assumes that the relative phases between the different components make the cross terms vanish on average, which is typically the case for broadband noise. The message here is that the phase noise PSD of the original pulse train is moved blockwise into the Nyquist range of the picked sequence and stacks up there as shown in Fig. 1(b). The integral of the PSD yields the mean square value of the phase error of the picked pulse train, so that

$$\langle\Phi_n^2\rangle = \frac{\Delta\omega}{2\pi}\sum_{l=0}^{M-1}\langle S_l^{\Phi}\rangle = \frac{\Delta\omega}{2\pi}\sum_{k=0}^{N-1}\langle S_k^{\psi}\rangle = \langle\psi_n^2\rangle, \tag{10}$$

and the rms value of the picked sequence fluctuations is the same as the rms value of the original pulse train up to the full bandwidth of $\omega_r/2$.

To demonstrate the above experimentally, we directly measured the PSD of CEP fluctuations with

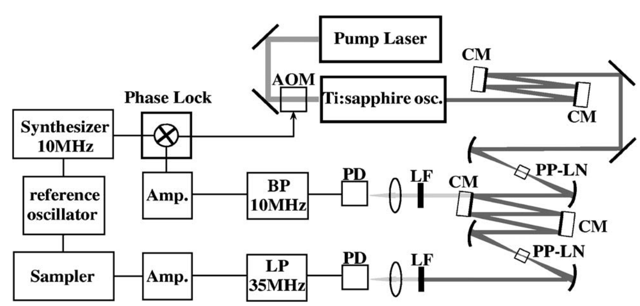

Fig. 2. Setup used to measure the CEP error of a simulated picked sequence: AOM, acousto-optic modulator; CM, chirped mirror; PP-LN, periodically poled MgO-doped lithium niobate; LF, long-pass filter; PD, photodiode; BP, bandpass filter; LP, low-pass filter; Amp., amplifier.

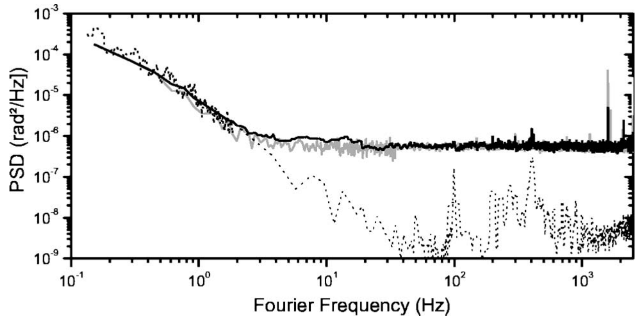

Fig. 3. PSD of phase fluctuation determined (gray) using the sampler at 5 kHz sampling frequency and 35 MHz bandwidth (black), using Eq. (9) and the full phase noise PSD, and (dotted) full PSD neglecting all components higher than the corresponding Nyquist frequency of 2.5 kHz.

the setup shown in Fig. 2, using CEP detection based on a periodically poled lithium niobate (PPLN) crystal described in more detail elsewhere.[9] The system consisted of two independent PPLN-CEP detectors. The first (in loop) was used to lock the carrier envelope offset frequency to a 10 MHz reference signal dervied from a low-noise quartz oscillator. With the second CEP detector (out of loop) we measured the CEP fluctuations Φ_n when locked. For this purpose we employed a 100 MHz digital sampler that was synchronized to a subharmonic at 5 kHz of the 10 MHz reference. In this way only pulses with the same average CEP were picked (sampled). The bandwidth of the sampler was adjusted to be identical with the Nyquist frequency $\omega_r/2$ so that there was always exactly one pulse from the laser present during each gate time of the sampler. For this reason it was not necessary to stabilize the repetition rate of the laser. At the same time this ensured that the full bandwidth of CEP noise was present in our data.

Since our PPLN-CEP detector delivers the sine of the CEP (up to an unknown phase offset) it can be adjusted to be maximally sensitive to phase variations and minimally sensitive to amplitude variations by choosing an appropriate reference phase in the phase-locked loop. The amplitude of the unlocked beat signal provides the voltage to phase conversion factor.

From the data obtained in this way, we calculated the left-hand side of Eq. (9). We measured the full PSD of the beat signal in the same way as described in Ref. 9 and calculated the right-hand side of Eq. (9). A comparison between these two data sets (Fig. 3) shows very good agreement in both magnitude and structure. The small discrepancy in the range from 0.5 to 20 Hz can be explained by different environmental conditions (air currents, etc.) that changed the quality of the lock during recording of various data sets. For reference we also plot the full PSD without taking aliasing into account.

In conclusion, we have shown that it is necessary to characterize the phase noise properties of a femtosecond oscillator up to the full bandwidth of $\omega_r/2$ to be able to derive the potential CEP stability of a low-repetition-rate chirped pulse amplification system seeded by this oscillator. The high-frequency components play an important role as demonstrated in Refs. 9 and 10. We have given an explicit formula for calculating the PSD of phase fluctuations of the amplified pulse train. Obviously, for estimating the stability of the output of the amplifier it is mandatory to also take noise sources inside the amplifier (such as beam pointing instabilities in the compressor) into account.[5] The analysis is also applicable to similar problems such as pump pulse synchronization in noncollinear optical parametric chirped-pulse amplification[11] systems. Finally, we performed a demonstration experiment to verify this formula in a real system, which showed nice agreement with our theoretical considerations.

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